FP Review

• Floating Point numbers approximate values that we want to use.
• IEEE 754 Floating Point Standard is most widely accepted attempt to standardize interpretation of such numbers.
  • Every desktop or server computer sold since ~1997 follows these conventions.
• Summary (single precision):

<table>
<thead>
<tr>
<th>S</th>
<th>Exponent</th>
<th>Significand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>8 bits</td>
<td>23 bits</td>
</tr>
</tbody>
</table>

-\((-1)^S\) x (1 + Significand) x \(2^{(\text{Exponent} - 127)}\)

• Double precision identical, bias of 1023

Representation for Denoms (1/3)

• Problem: There's a gap among representable FP numbers around 0
  • Smallest representable pos num:
    \[ a = 1.0...02^{-126} = 2^{-126} \]
  • Second smallest representable pos num:
    \[ b = 1.000...12^{-126} = 2^{-126} + 2^{-149} \]
  \( a - 0 = 2^{-126} \)
  \( b - a = 2^{-149} \)

Gaps!

Normalization and implicit 1 is to blame!

representation for Denoms (2/3)

• Solution:
  • We still haven't used Exponent = 0, Significand nonzero
  • Denormalized number: no leading 1, implicit exponent = -126.
  • Smallest representable pos num:
    \[ a = 2^{-149} \]
  • Second smallest representable pos num:
    \[ b = 2^{-148} \]

IEEE Four Rounding Modes

• Math on real numbers \(\Rightarrow\) we worry about rounding to fit result in the significant field.
• FP hardware carries 2 extra bits of precision, and rounds for proper value.
• Rounding occurs when converting…
  • double to single precision
  • floating point # to an integer
IEEE Four Rounding Modes

- Round towards $+\infty$
  - ALWAYS round “up”: $2.1 \Rightarrow 3$, $-2.1 \Rightarrow -2$
- Round towards $-\infty$
  - ALWAYS round “down”: $1.9 \Rightarrow 1$, $-1.9 \Rightarrow -2$
- Truncate
  - Just drop the last bits (round towards 0)
- Round to (nearest) even (default)
  - Normal rounding, almost: $2.5 \Rightarrow 2$, $3.5 \Rightarrow 4$
  - Like you learned in grade school
  - Insures fairness on calculation
  - Half the time we round up, other half down

Clarification - IEEE Four Rounding Modes

- This is just an example in base 10 to show you the 4 modes.
- What really happens is...
  1) in binary, not decimal!
  2) at the lowest bit of the mantissa with the guard bit(s) as our extra bit(s), and you need to decide how these extra bit(s) affect the result if the guard bits are “100…”
  3) if so, you’re half-way between the representable numbers.
  E.g., $0.1010$ is $5/8$, halfway between our representable $4/8$ [$1/2$] and $6/8$ [$3/4$]. Which number do we round to? 4 modes!

Integer Multiplication (1/3)

- Paper and pencil example (unsigned):
  
  | Multiplicand | 1000 | 8 |
  | Multiplier   | x1001 | 9 |
  | 1000         | 0000  |
  | 0000         | +1000 |
  | 01001000     |       |

  - $m$ bits x $n$ bits = $m + n$ bit product

Integer Multiplication (2/3)

- In MIPS, we multiply registers, so:
  - 32-bit value x 32-bit value = 64-bit value
- Syntax of Multiplication (signed):
  - `mult register1, register2`
    - Multiplies 32-bit values in those registers & puts 64-bit product in special result regs:
      - puts product upper half in `hi`, lower half in `lo`
    - `hi` and `lo` are 2 registers separate from the 32 general purpose registers
    - Use `mfhi register` & `mflo register` to move from `hi`, `lo` to another register

Integer Multiplication (3/3)

- Example:
  - in C: `a = b * c;`
  - in MIPS:
    - `mult $s2,$s3` # b*c
    - `mfhi $s0` # upper half of
      - `product into $s0`
    - `mflo $s1` # lower half of
      - `product into $s1`

  - Note: Often, we only care about the lower half of the product.

Integer Division (1/2)

- Paper and pencil example (unsigned):
  | Quotient | 1001 |
  | Divisor  | 10001010 |
  | Dividend | -1000 |

  - 10
  - 101
  - 1010
  - -1000

  - 10 Remainder
    - (or Modulo result)
  - Dividend = Quotient x Divisor + Remainder
**Integer Division (2/2)**

- **Syntax of Division (signed):**
  - `div register1, register2`
  - Divides 32-bit register 1 by 32-bit register 2:
  - Puts remainder of division in `hi`, quotient in `lo`
- **Implements C division (`/`) and modulo (`%`)**
- **Example in C:**
  ```c
  a = c / d;
  b = c % d;
  ```
- **MIPS:**
  ```mips
  $s0 ↔ a; $s1 ↔ b; $s2 ↔ c; $s3 ↔ d;
  div $s2, $s3 # lo=c/d, hi=c%d
  mflo $s0 # get quotient
  mfhi $s1 # get remainder
  ```

**Unsigned Instructions & Overflow**

- **MIPS also has versions of `mult`, `div` for unsigned operands:**
  ```mips
  multu
  divu
  ```
- **Determines whether or not the product and quotient are changed if the operands are signed or unsigned.**
- **MIPS does not check overflow on ANY signed/unsigned multiply, divide instr**
- **Up to the software to check `hi`**

**FP Addition & Subtraction**

- **Much more difficult than with integers (can’t just add significands)**
- **How do we do it?**
  - De-normalize to match larger exponent
  - Add significands to get resulting one
  - Normalize (& check for under/overflow)
  - Round if needed (may need to renormalize)
- **If signs ≠, do a subtract. (Subtract similar)**
  ```mips
  if signs # for add (or = for sub), what’s ans sign?
  ```
- **Question: How do we integrate this into the integer arithmetic unit? [Answer: We don’t!]**

**MIPS Floating Point Architecture (1/4)**

- **Separate floating point instructions:**
  - Single Precision:
    ```mips
    add.s, sub.s, mul.s, div.s
    ```
  - Double Precision:
    ```mips
    add.d, sub.d, mul.d, div.d
    ```
- **These are far more complicated than their integer counterparts**
- **Can take much longer to execute**

**MIPS Floating Point Architecture (2/4)**

- **Problems:**
  - Inefficient to have different instructions take vastly differing amounts of time.
  - Generally, a particular piece of data will not change FP within a program.
    - Only 1 type of instruction will be used on it.
  - Some programs do no FP calculations
  - It takes lots of hardware relative to integers to do FP fast

**MIPS Floating Point Architecture (3/4)**

- **1990 Solution: Make a completely separate chip that handles only FP.**
- **Coprocessor 1:**
  - **FP chip**
    ```mips
    contains 32 32-bit registers: $f0, $f1, ...
    ```
  - **most of the registers specified in .s and .d instruction refer to this set**
    - Only 1 type of instruction will be used on it.
  - **Some programs do no FP calculations**
  - **It takes lots of hardware relative to integers to do FP fast**
  - **Double Precision: by convention, even/odd pair contain one DP FP number:**
    ```mips
    $f0/$f1, $f2/$f3, ..., $f30/$f31
    ```
    - **Even register is the name**
MIPS Floating Point Architecture (4/4)

• 1990 Computer actually contains multiple separate chips:
  • Processor: handles all the normal stuff
  • Coprocessor 1: handles FP and only FP;
  • more coprocessors?... Yes, later
  • Today, FP coprocessor integrated with CPU, or cheap chips may leave out FP HW
• Instructions to move data between main processor and coprocessors:
  • mfc0, mtc0, mfc1, mtc1, etc.
• Appendix pages A-70 to A-74 contain many, many more FP operations.

FP/Math Summary

• Reserve exponents, significands:
<table>
<thead>
<tr>
<th>Exponent</th>
<th>Significand</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>nonzero</td>
<td>Denorm</td>
</tr>
<tr>
<td>1-254</td>
<td>anything</td>
<td>+/- fl. pt. #</td>
</tr>
<tr>
<td>255</td>
<td>0</td>
<td>+/- inf</td>
</tr>
<tr>
<td>255</td>
<td>nonzero</td>
<td>NaN</td>
</tr>
</tbody>
</table>
• Integer mult, div uses hi, lo regs
  • mfhi and mflo copies out.
• Four rounding modes (to even default)
  • MIPS FL ops complicated, expensive

Administrivia

• Midterm TOMORROW!!!11!one!
  • 11:00 – 2:00
  • 277 Cory
  • You may bring with you:
    - The green sheet from COD or a photocopy thereof
    - One 8 ½” x 11” note sheet with handwritten notes on one side
    - No books, calculators, other shenanigans
  • Conflicts, DSP, other issues:
    - let me know ASAP
  • Project 1 is due Sunday night

Review from before: lui

• So how does lui help us?
  • Example:
    addi $t0,$t0, 0xABABCDCD
    becomes:
    lui $at, 0xABAB
    ori $at, $at, 0xCDCCD
    add $t0,$t0,$at
  • Now each I-format instruction has only a 16-bit immediate.
  • Wouldn't it be nice if the assembler would do this for us automatically?
    - If number too big, then just automatically replace addi with lui, ori, add

True Assembly Language (1/3)

• Pseudoinstruction: A MIPS instruction that doesn't turn directly into a machine language instruction, but into other MIPS instructions
• What happens with pseudoinstructions?
  • They're broken up by the assembler into several "real" MIPS instructions.
  • But what is a "real" MIPS instruction? Answer in a few slides
• First some examples

Example Pseudoinstructions

• Register Move
  move reg2,reg1
  Expands to:
  add reg2,$zero,reg1
• Load Immediate
  li reg,value
  If value fits in 16 bits:
  addi reg,$zero,value
  else:
  lui reg,upper 16 bits of value
  ori reg,$zero,lower 16 bits
True Assembly Language (2/3)

• Problem:
  • When breaking up a pseudoinstruction, the assembler may need to use an extra reg.
  • If it uses any regular register, it’ll overwrite whatever the program has put into it.
• Solution:
  • Reserve a register ($1, called $at for “assembler temporary”) that assembler will use to break up pseudo-instructions.
  • Since the assembler may use this at any time, it’s not safe to code with it.

Example Pseudoinstructions

• Rotate Right Instruction
  ror reg, value

  Expands to:
  srl $at, reg, value
  sll reg, reg, 32-value
  or reg, reg, $at
• “No Operation” instruction
  nop

  Expands to instruction = 0ten,
  sll $0, $0, 0

Example Pseudoinstructions

• Wrong operation for operand
  addu reg,reg,value # should be addiu

  If value fits in 16 bits, addiu is changed to:
  addiu reg,reg,value
  else:
  lui $at,upper 16 bits of value
  ori $at,$at,lower 16 bits
  addu reg,reg,$at

• How do we avoid confusion about whether we are talking about MIPS assembler with or without pseudoinstructions?

True Assembly Language (3/3)

• MAL (MIPS Assembly Language): the set of instructions that a programmer may use to code in MIPS; this includes pseudoinstructions
• TAL (True Assembly Language): set of instructions that can actually get translated into a single machine language instruction (32-bit binary string)

• A program must be converted from MAL into TAL before translation into 1s & 0s.

Questions on Pseudoinstructions

• Question:
  • How does MIPS recognize pseudoinstructions?
• Answer:
  • It looks for officially defined pseudoinstructions, such as ror and move
  • It looks for special cases where the operand is incorrect for the operation and tries to handle it gracefully

Rewrite TAL as MAL

• TAL:

  Loop:  
or $v0,$0,$0
  slt $t0,$0,$a1
  beq $t0,$0,Exit
  add $v0,$v0,$a0
  addi $a1,$a1,-1
  j Loop

  Exit:

  • This time convert to MAL
  • It’s OK for this exercise to make up MAL instructions
### Rewrite TAL as MAL (Answer)

- **TAL:**
  ```
  Loop:  or $v0,$0,$0
         s1t $t0,$0,$a1
         beq $t0,$0,Exit
         add $v0,$v0,$a0
         addi $a1,$a1,-1
         j Loop
  Exit:
  ```
- **MAL:**
  ```
  li $v0,0
  Loop:  bge $zero,$a1,Exit
         add $v0,$v0,$a0
         sub $a1,$a1,1
         j Loop
  Exit:
  ```

### Peer Instruction

Which of the instructions below are MAL and which are TAL?

- A. `addi $t0, $t1, 40000`
- B. `beq $s0, 10, Exit`
- C. `sub $t0, $t1, 1`

### In conclusion

- Assembler expands real instruction set (TAL) with pseudoinstructions (MAL)
  - Only TAL can be converted to raw binary
  - Assembler's job to do conversion
  - Assembler uses reserved register `$at`
  - MAL makes it much easier to write MIPS