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CS61C : Machine Structures

Lecture #10 – Instruction Representation II, Floating Point I

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Review…

- Logical and Shift Instructions
  - Operate on individual bits (arithmetic operate on entire word)
  - Use to isolate fields, either by masking or by shifting back & forth
  - Use shift left logical, `sll`, for multiplication by powers of 2
  - Use shift right arithmetic, `sra`, for division by powers of 2
- Simplifying MIPS: Define instructions to be same size as data word (one word) so that they can use the same memory (compiler can use `lw` and `sw`).
- Computer actually stores programs as a series of these 32-bit numbers.
- MIPS Machine Language Instruction: 32 bits representing a single instruction

R	opcode	rs	rt	rd	shamt	funct
I	opcode	rs	rt	immediate		
J	opcode	target address				

I-Format Problems (0/3)

- Problem 0: Unsigned # sign-extended?
  - `addiu`, `sltiu`, sign-extends immediates to 32 bits. Thus, # is a "signed" integer.
- Rationale
  - `addiu` so that can add w/out overflow
    - See K&R pp. 230, 305
  - `sltiu` suffers so that we can have ez HW
    - Does this mean we'll get wrong answers?
    - Nope, it means assembler has to handle any unsigned immediate $2^{15} \le n < 2^{16}$ (I.e., with a 1 in the 15th bit and 0s in the upper 2 bytes) as it does for numbers that are too large. ⇒

I-Format Problems (1/3)

- Problem 1:
  - Chances are that `addi`, `lw`, `sw` and `slti` will use immediates small enough to fit in the immediate field.
  - …but what if it's too big?
  - We need a way to deal with a 32-bit immediate in any I-format instruction.

I-Format Problems (2/3)

- Solution to Problem 1:
  - Handle it in software + new instruction
  - Don't change the current instructions: instead, add a new instruction to help out
- New instruction:

```
lui   register, immediate
```

  - stands for Load Upper Immediate
  - takes 16-bit immediate and puts these bits in the upper half (high order half) of the specified register
  - sets lower half to 0s

I-Format Problems (3/3)

- Solution to Problem 1 (continued):
  - So how does `lui` help us?
  - Example:

```
addi   $t0,$t0, 0xABABCDCD
```

becomes:

```
lui    $at, 0xABAB
ori    $at, $at, 0xCDCD
add    $t0,$t0,$at
```

  - Now each I-format instruction has only a 16-bit immediate.
  - Wouldn't it be nice if the assembler would this for us automatically? (later)

Branches: PC-Relative Addressing (1/5)

- Use I-Format

opcode	rs	rt	immediate

- opcode specifies beq v. bne
- rs and rt specify registers to compare
- What can immediate specify?
  - Immediate is only 16 bits
  - PC (Program Counter) has byte address of current instruction being executed; 32-bit pointer to memory
  - So immediate cannot specify entire address to branch to.

Branches: PC-Relative Addressing (2/5)

- How do we usually use branches?
  - Answer: if-else, while, for
  - Loops are generally small: typically up to 50 instructions
  - Function calls and unconditional jumps are done using jump instructions (j and jal), not the branches.
- Conclusion: may want to branch to anywhere in memory, but a branch often changes PC by a small amount

Branches: PC-Relative Addressing (3/5)

- Solution to branches in a 32-bit instruction: PC-Relative Addressing
- Let the 16-bit immediate field be a signed two's complement integer to be *added* to the PC if we take the branch.
- Now we can branch ± 2^{15} bytes from the PC, which should be enough to cover almost any loop.
- Any ideas to further optimize this?

Branches: PC-Relative Addressing (4/5)

- Note: Instructions are words, so they're word aligned (byte address is always a multiple of 4, which means it ends with 00 in binary).
  - So the number of bytes to add to the PC will always be a multiple of 4.
  - So specify the immediate in words.
- Now, we can branch ± 2^{15} words from the PC (or ± 2^{17} bytes), so we can handle loops 4 times as large.

Branches: PC-Relative Addressing (5/5)

- Branch Calculation:
  - If we don't take the branch:

    PC = PC + 4

    PC+4 = byte address of next instruction
  - If we do take the branch:

    PC = (PC + 4) + (immediate * 4)
  - Observations
    - Immediate field specifies the number of words to jump, which is simply the number of instructions to jump.
    - Immediate field can be positive or negative.
    - Due to hardware, add immediate to (PC+4), not to PC; will be clearer why later in course

Branch Example (1/3)

- MIPS Code:

```
Loop: beq  $9,$0,End
      add  $8,$8,$10
      addi $9,$9,-1
      j    Loop
End:
```

- beq branch is I-Format:

  opcode = 4 (look up in table)

  rs = 9 (first operand)

  rt = 0 (second operand)

  immediate = ???

Branch Example (2/3)

- **MIPS Code:**

```
Loop: beq   $9,$0,End
      addi  $8,$8,$10
      addi  $9,$9,-1
      j     Loop
End:
```

- **`Immediate` Field:**
  - Number of instructions to add to (or subtract from) the PC, starting at the instruction *following* the branch.
  - In `beq` case, `immediate` = 3

Branch Example (3/3)

- **MIPS Code:**

```
Loop: beq   $9,$0,End
      addi  $8,$8,$10
      addi  $9,$9,-1
      j     Loop
End:
```

decimal representation:

4	9	0	3

binary representation:

000100	01001	00000	0000000000000011

Questions on PC-addressing

- Does the value in branch field change if we move the code?
- What do we do if destination is > 2^{15} instructions away from branch?
- Since it's limited to ± 2^{15} instructions, doesn't this generate lots of extra MIPS instructions?
- Why do we need all these addressing modes? Why not just one?

Green Sheet Errors

- Section 1: The Core Instruction Set
  - lb, lbu, lw scratch out 0/
  - sll, srl shift rt not rs so change R[rs] to R[rt]
  - jal should be R[31] = PC + 8, not +4
- Section 2: Register Name, Number, Use, Call Convention
  - $ra is not preserved across calls so make yes a no

J-Format Instructions (1/5)

- For branches, we assumed that we won't want to branch too far, so we can specify *change* in PC.
- For general jumps (`j` and `jal`), we may jump to *anywhere* in memory.
- Ideally, we could specify a 32-bit memory address to jump to.
- Unfortunately, we can't fit both a 6-bit opcode and a 32-bit address into a single 32-bit word, so we compromise.

J-Format Instructions (2/5)

- Define "fields" of the following number of bits each:

6 bits	26 bits

- As usual, each field has a name:

opcode	target address

- Key Concepts
  - Keep `opcode` field identical to R-format and I-format for consistency.
  - Combine all other fields to make room for large target address.

J-Format Instructions (3/5)

- For now, we can specify 26 bits of the 32-bit bit address.
- Optimization:
  - Note that, just like with branches, jumps will only jump to word aligned addresses, so last two bits are always 00 (in binary).
  - So let's just take this for granted and not even specify them.

J-Format Instructions (4/5)

- Now specify 28 bits of a 32-bit address
- Where do we get the other 4 bits?
  - By definition, take the 4 highest order bits from the PC.
  - Technically, this means that we cannot jump to *anywhere* in memory, but it's adequate 99.9999…% of the time, since programs aren't that long
    - only if straddle a 256 MB boundary
  - If we absolutely need to specify a 32-bit address, we can always put it in a register and use the `jr` instruction.

J-Format Instructions (5/5)

- Summary:
  - New PC = { PC[31..28], target address, 00 }
- Understand where each part came from!
- Note: { , , } means concatenation { 4 bits , 26 bits , 2 bits } = 32 bit address
  - { 1010, 11111111111111111111111111, 00 } = 10101111111111111111111111111100
  - Note: Book uses ||

In semi-conclusion…

- MIPS Machine Language Instruction: 32 bits representing a single instruction

R	opcode	rs	rt	rd	shamt	funct
I	opcode	rs	rt	immediate		
J	opcode	target address				

- Branches use PC-relative addressing, Jumps use absolute addressing.
- Disassembly is simple and starts by decoding `opcode` field. (more in a week)

Review of Numbers

- Computers are made to deal with numbers
- What can we represent in N bits?
  - Unsigned integers:

    0 to $2^N - 1$
  - Signed Integers (Two's Complement)

    $-2^{(N-1)}$ to $2^{(N-1)} - 1$

Other Numbers

- What about other numbers?
  - Very large numbers? (seconds/century) $3,155,760,000_{10}$ (3.15576_{10} x 10^9)
  - Very small numbers? (atomic diameter) 0.00000001_{10} (1.0_{10} x 10^{-8})
  - Rationals (repeating pattern) 2/3 (0.666666666. . .)
  - Irrationals $2^{1/2}$ (1.414213562373. . .)
  - Transcendentals *e* (2.718...), π (3.141...)
- All represented in scientific notation

Scientific Notation (in Decimal)

mantissa → $6.02_{10} \times 10^{23}$ ← exponent

decimal point; radix (base)

- Normalized form: no leadings 0s (exactly one digit to left of decimal point)
- Alternatives to representing 1/1,000,000,000
  - Normalized: 1.0×10^{-9}
  - Not normalized: $0.1 \times 10^{-8}, 10.0 \times 10^{-10}$

Scientific Notation (in Binary)

mantissa → $1.0_{two} \times 2^{-1}$ ← exponent

"binary point"; radix (base)

- Computer arithmetic that supports it called floating point, because it represents numbers where the binary point is not fixed, as it is for integers
  - Declare such variable in C as `float`

Floating Point Representation (1/2)

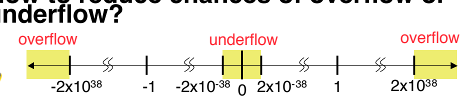
- Normal format: $+1.xxxxxxxxxx_{two} * 2^{yyyy_{two}}$
- Multiple of Word Size (32 bits)

31	30 … 23	22 … 0
S	Exponent	Significand
1 bit	8 bits	23 bits

- S represents Sign
  Exponent represents y's
  Significand represents x's
- Represent numbers as small as 2.0×10^{-38} to as large as 2.0×10^{38}

Floating Point Representation (2/2)

- What if result too large? ($> 2.0 \times 10^{38}$)
  - Overflow!
  - Overflow ⇒ Exponent larger than represented in 8-bit Exponent field
- What if result too small? ($>0, < 2.0 \times 10^{-38}$)
  - Underflow!
  - Underflow ⇒ Negative exponent larger than represented in 8-bit Exponent field
- How to reduce chances of overflow or underflow?



Double Precision Fl. Pt. Representation

- Next Multiple of Word Size (64 bits)

31	30 … 20	19 … 0
S	Exponent	Significand
1 bit	11 bits	20 bits

Significand (cont'd)
32 bits

- Double Precision (vs. Single Precision)
  - C variable declared as `double`
  - Represent numbers almost as small as 2.0×10^{-308} to almost as large as 2.0×10^{308}
  - But primary advantage is greater accuracy due to larger significand

QUAD Precision Fl. Pt. Representation

- Next Multiple of Word Size (128 bits)
- Unbelievable range of numbers
- Unbelievable precision (accuracy)
- This is currently being worked on
- The current version has 15 bits for the exponent and 112 bits for the significand
- Oct-Precision? It's been implemented before… (256 bit)
- Half-Precision? Yep, that's for a short (16 bit)

IEEE 754 Floating Point Standard (1/4)

- Single Precision, DP similar
- Sign bit: 1 means negative
  0 means positive
- Significand:
  - To pack more bits, leading 1 implicit for normalized numbers
  - 1 + 23 bits single, 1 + 52 bits double
  - always true: 0 < Significand < 1
    (for normalized numbers)
- Note: 0 has no leading 1, so reserve exponent value 0 just for number 0

IEEE 754 Floating Point Standard (2/4)

- Kahan wanted FP numbers to be used even if no FP hardware; e.g., sort records with FP numbers using integer compares
- Could break FP number into 3 parts: compare signs, then compare exponents, then compare significands
- Wanted it to be faster, single compare if possible, especially if positive numbers
- Then want order:
  - Highest order bit is sign (negative < positive)
  - Exponent next, so big exponent => bigger #
  - Significand last: exponents same => bigger #

IEEE 754 Floating Point Standard (3/4)

- Negative Exponent?
  - 2's comp? 1.0 x 2^{-1} v. 1.0 x2^{+1} (1/2 v. 2)

1/2	0	1111 1111	000 0000 0000 0000 0000 0000
2	0	0000 0001	000 0000 0000 0000 0000 0000

  - This notation using integer compare of 1/2 v. 2 makes 1/2 > 2!
- Instead, pick notation 0000 0001 is most negative, and 1111 1111 is most positive
  - 1.0 x 2^{-1} v. 1.0 x2^{+1} (1/2 v. 2)

1/2	0	0111 1110	000 0000 0000 0000 0000 0000
2	0	1000 0000	000 0000 0000 0000 0000 0000

IEEE 754 Floating Point Standard (4/4)

- Called Biased Notation, where bias is number subtract to get real number
  - IEEE 754 uses bias of 127 for single prec.
  - Subtract 127 from Exponent field to get actual value for exponent
  - 1023 is bias for double precision
- Summary (single precision):

31	30 … 23	22 … 0
S	Exponent	Significand
1 bit	8 bits	23 bits

- $(-1)^S$ x (1 + Significand) x $2^{(\text{Exponent-127})}$
  - Double precision identical, except with exponent bias of 1023

Peer Instruction

1	1000 0001	111 0000 0000 0000 0000 0000

What is the decimal equivalent of the floating pt # above?

```
1: -1.75
2: -3.5
3: -3.75
4: -7
5: -7.5
6: -15
7: -7 * 2^129
8: -129 * 2^7
```


"And in conclusion…"

- Floating Point numbers approximate values that we want to use.
- IEEE 754 Floating Point Standard is most widely accepted attempt to standardize interpretation of such numbers
  - Every desktop or server computer sold since ~1997 follows these conventions
- Summary (single precision):

31	30 … 23	22 … 0
S	Exponent	Significand
1 bit	8 bits	23 bits

- $(-1)^S$ x (1 + Significand) x $2^{(\text{Exponent-127})}$
  - Double precision identical, bias of 1023