Graphs and Eulerian Tours.

Euler 1735

Königsberg.

Abstraction:

\[ \{A, B, C, D\} \text{ vertices.} \]
\[ \{A, B\}, \{B, D\}, \{B, D\} \text{ edges.} \]
Google
Web graph
Page rank
Web graph
EU Buys Large Graph for 1.2 Billion Euros.

Connecting pattern of the brain:

Evolution: Phylogeny trees.
$G(V, E)$ graph $G$ with vertex set $V$
and edge set $E$.

$V = \{1, 2, 3, 4, 5, 6\}$

$U \times W = \{(u, w) : u \in U \land w \in W\}$
$U = \{1, 2, 3\}$
$W = \{a, b\}$
$U \times W = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$
$E \subseteq V \times V$
$E = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
$E = \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$
$\text{deg}(6) = 5$
$\text{deg}(1) = 3$
$\text{deg}(3) = 5$
$\text{deg}(4) = 2$

Directed

Undirected
in-deg(6) = 2
out-deg(6) = 3

Simple cycle

\[ \sum_{v \in V} \deg(v) = ? = 2 |E| \]

Cor. Number of odd degree vertices in any graph \(G(V, E)\) must be even.

If vertex \(v\) has no neighbors i.e. \(\deg(v) = 0\) the \(v\) is called an isolated vertex.

Path: 1, 2, 3, 6, 5
Simple path. no repetitions.

Walk: 1, 2, 3, 1, 6, 3, 5

Simple cycle: 1, 2, 3, 6, 1

Tour: 1, 2, 6, 1, 3, 5, 6, 1
**Theorem**: A graph has an Eulerian tour iff it is connected (up to isolated vertices) and every vertex has even degree.

**Idea**: every time you enter a vertex it must leave. Each such visit uses up two edges at its total degree.

\[ \Rightarrow \]

leave every true vertex has even degree

\[ \Leftarrow \]

start
Given: connected graph (up to isolated vertices) with all vertices even degree.

Start roughly from one vertex. Can't get stuck anywhere except if you return to starting point.
when return to only stuck at starty pt:

only two odd degree vertices in tour.

Already removed black part.

All other edges are connected to black part of graph & have even degree.