Counting

\[ S = \{1, 2, \ldots, n\} \quad n = 52. \]

1. Deal a sequence \( f_k = 5 \).
   Order matters. \( 1, 2, 3, 4, 5 \)
   \( \neq 2, 1, 3, 4, 5 \).

1st Rule of count: Create an object by sequence of choices.
\( n_1 \) ways of making 1st choice, no matter what we chose
\( n_2 \) " 2nd choice, " " " " "
\( n_k \) " kth choice.

Total # distinct objects \( n_1 \times n_2 \ldots \times n_k \).

\[ n(n-1)(n-2)\ldots(n-k+1) = \frac{n!}{(n-k)!}. \]

\[ 52 \times 51 \times 50 \times 49 \times 48 \approx 312 \text{ Million} \]
$S = \xi_1, \ldots, \xi_n$ 

$n = 52.$

2. Pick a subset of $K$ elements.

\[ K=1 \quad n \text{ ways.} \]
\[ K=2 \]
\[ \frac{5 \cdot 2}{2} = 5 \]
\[ \frac{n(n-1)}{2} \]
\[ K=3 \]
\[ \frac{5 \cdot 2 \cdot 1}{2} = 5 \]
\[ \frac{n(n-1)(n-2)}{6} \]

2nd Rule of Counting.
# poker hands as an uct = \binom{52}{5}

= \frac{52!}{47! \cdot 5!} \approx 2.6 \text{ Million}

# ordered poker hands = \frac{52!}{47!} = 52 \times 51 \times 50 \times 49 \times 48

\approx 312 \text{ Million}

5! = 5 \times 4 \times 3 \times 2 = 120
$S = \{1, \ldots , n\}$

Pick a $k$-subset. [order does not matter]

1. Pretend order matters:

$$n(n-1)(n-2)\ldots(n-k+1) \equiv \frac{n!}{(n-k)!}$$

2. How many ordered objects / unordered object?

   How many $k$-sequences / $k$-set?\text{?}

   \$1, 2, \ldots , k \in S\$. How many ways of order them?

   \$k(k-1)\ldots2\cdot1 = k!\$

3. # unordered objects = # $k$-subsets

   \[
   \frac{\#\text{ordered objects}}{\#\text{ordered objects}/\text{unordered object}}
   \]

   \[
   = \frac{n!}{(n-k)!k!} = \binom{n}{k} = \binom{n}{k}
   \]

   \$n\choose k$.
\[(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2.\]
\[(x+y)^3 = (x+y)(x+y)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3.\]
\[(x+y)^n = 1 \times x^n + \binom{n}{1} x^{n-1} y + \frac{n(n-1)}{2} x^{n-2} y^2 + \ldots + \binom{n}{n-k} x^{n-k} y^k + \ldots + x y^{n-1} + y^n\]

\[= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k\]

Substitute \(x = y = 1\).

\[2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n-1} + \binom{n}{n}.\]
$2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$.

$S = \sum_{k=1}^{n} k$.

How many subsets of $S$ are there?

$\sum_{k=0}^{n} \# \text{ of } k \text{-subsets}$

$= \sum_{k=0}^{n} \binom{n}{k}$.

Sequence of choices:
- put 1 in subset or not
- put 2 in subset or not
-...
- put $n$ in subset or not.

$1^{st}$ rule of counting:

$\underbrace{2 \times 2 \times 2 \ldots \times 2}_{n \text{ times}} = 2^n$. 
\[ S = \{31, \ldots, \eta^3\} \quad n = 6 \]
\[ n = \frac{2^6}{3} \]

1. Sequence of \( K \) (ordered)

2. Subset of \( K \) (unordered)

\[ 1, 1, 2, 1, 5 = 3 \{3\}, 1\{2\}, 1\{5\} \]

\[ \frac{n}{n_1 n_2 \cdots n_K} = n^K \]

Coin \( 2^K \) possible outcomes.

Dice \( 6^K \) possible outcomes.

\[ K = 2 \quad 6^2 = 36 \]

Sum of two dice = \( m \).

\begin{align*}
\text{Sum of two dice} &= m \\
\quad m = 2 &\quad \boxed{\text{\(\square\) \(\square\)}} \\
\quad m = 3 &\quad \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}} \\
\quad m = 7 &\quad \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}}} \\
\quad m = 8 &\quad \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}, \boxed{\text{\(\square\) \(\square\)}}} \\
\end{align*}

\[ 1 \text{ choice:} \quad \boxed{6} \quad \sqrt{\quad \text{sum} = m} \quad \boxed{6 \times 1 = 6} \]
sample with replacement when order does not matter.

\[ S = \{1, \ldots, n^3\} \]

1st rule: \[ k \] elements.

order does not matter

2nd Rule: 4 tech

3 tech 1 hum

How many schedules.

4 cases.

(tech, tech, tech, tech)

(tech, tech, tech, tech)

(humanities, tech, tech, tech)

(humanities, hum, hum, hum)

(humanities, hum, hum, hum)
n choose k.

\[ \binom{6}{2} \]
$S = \{ x_1, \ldots, x_n \}$.

Pick $K$ elements.

$n\choose k$ walls for boxes = 1's.

$k$ theems. = 0's.

$n + k - 1$ length binary string.
with $k$ 0's.

$\binom{n + k - 1}{k}$