CS 70
Fall 2013

Counting

$S = \{1, 2, \ldots, n\}$ $\quad n = 52$.

1. Deal a sequence of $k = 5$.
Order matters. $\quad 1, 2, 3, 4, 5$
$\neq 2, 1, 3, 4, 5$.

1st Rule of counting: Create an object by sequence of choices.
n_1 ways of making 1st choice, no matter what we chose
n_2 " " " 2nd choice, " " " " "
n_3 " " " 3rd choice, - - - - - -
⋮
n_k " " " kth choice.
Then # distinct objects is $n_1 n_2 \cdots n_k$.

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

$52 \times 51 \times 50 \times 49 \times 48 \approx 312$ Million

$S = \{1, \ldots, n\}$ $\qquad n = 52.$

2. Pick a subset of k elements.

$k=1$ $\qquad n$ ways.

$\underline{\underline{k=2}}$ $\qquad \underline{5}\ \underline{2}$ $\qquad \dfrac{n(n-1)}{2}$

$\underline{2}\ \underline{5}$

$\underline{\underline{k=3}}$ $\qquad \underline{5}\ \underline{2}\ \underline{1}$ $\qquad \dfrac{n(n-1)(n-2)}{6}$

$\underline{2}\ \underline{5}\ \underline{1}$

$\underline{2}\ \underline{1}\ \underline{5}$

$\underline{1}\ \underline{2}\ \underline{5}$

$\underline{1}\ \underline{5}\ \underline{2}$

$\underline{5}\ \underline{1}\ \underline{2}$

<u>2nd Rule of Counting</u>:

poker hands as a set $= \binom{52}{5}$

$$= \frac{52!}{47!\,5!} \approx 2.6 \text{ Million.}$$

ordered poker hands $= \dfrac{52!}{47!} = 52 \times 51 \times 50 \times 49 \times 48$

≈ 312 Million.

$5! = 5 \times 4 \times 3 \times 2 = 120.$

$S = \{1, \ldots, n\}$.

Pick a k-subset. [order does not matter].

1. ~~Pretend~~ Pretend order matters:

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

2. How many ordered objects / unordered object?

How many k-sequences / k-~~set~~ subset?

$\{1, 2, \ldots, k\}$. How many ways of ordering them?

$$k(k-1)\cdots 2\cdot 1 = k!$$

3. # unordered objects = # k-subsets

$$= \frac{\#\text{ ordered objects}}{\#\text{ ordered objects / unordered object}}$$

$$= \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k} \text{ n choose } k.$$

$$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2.$$

$$(x+y)^3 = (x+y)(x+y)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3.$$

$$(x+y)^n = 1\,x^n + n\,x^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k$$

$$+ \_\, xy^{n-1} + y^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Substitute $x = y = 1$.

$$2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n-1} + \binom{n}{n}.$$

$$2^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}.$$

$$S = \{1, 2, 3, \ldots, n\}.$$

$0\ 1,\ 0 \ldots\ 1$

How many subsets of S are there?

$$\sum_{k=0}^{n} \# \text{ of } k\text{-subsets}.$$

$$= \sum_{k=0}^{n} \binom{n}{k}.$$

Sequence of choices:

put 1 in subset or not

put 2 in subset or not

$\vdots$

put n in subset or not.

1st rule of counting

$$\overbrace{2 \times 2 \times 2 \cdots \times 2}^{n \text{ times}} = 2^n.$$

with replacement.

$S = \{1, \dots, n\}$.

$n = 6$

$n = 2$

1. Sequence of k (ordered)
2. Subsets of k (unordered).

$1,1,2,1,5 = 3$ 1's, 1 2, 1 5

$$\underbrace{\underline{n}\ \underline{n}\ \underline{n}\ \underline{n}\ \underline{n}}_{k} = n^k.$$

Coin 2^k possible outcomes.

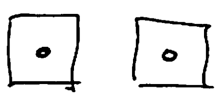
Dice 6^k possible outcomes.

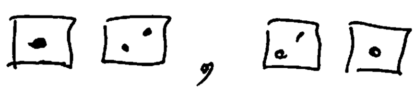
$k=2 \qquad 6^2 = 36$

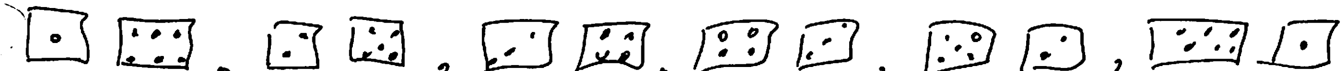
Sum of two dice $= m$.

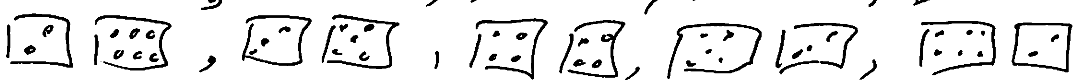
$\underline{6}\ \underline{}$ 1 choice: sum $= m$.

$6 \times 1 = 6$.

$m=2$ 

$m=3$ 

$m=7$ 

$m=8$ 

sampling with replacement when order does not matter.

$S = \{1, \ldots, n\}$

K elements.

$S = \{$technical, humanities, fun$\}$.

4 courses.

How many schedules.

order does not matter

$\updownarrow$

1st rule $\times$.

2nd Rule:

4 tech

3 tech, 1 hum

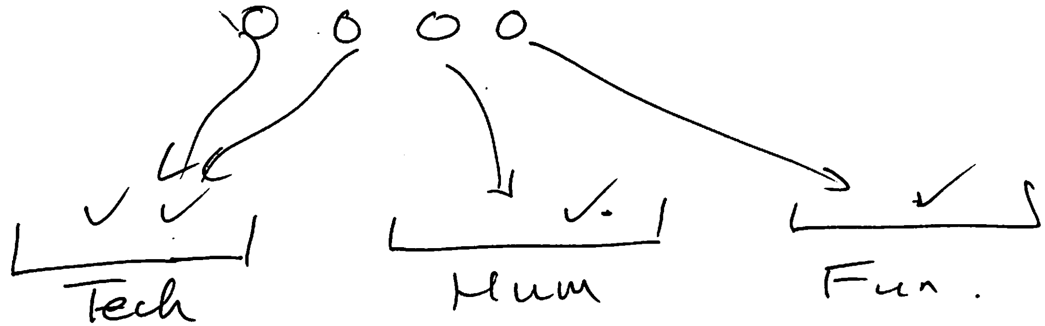
(tech, tech, tech, tech).

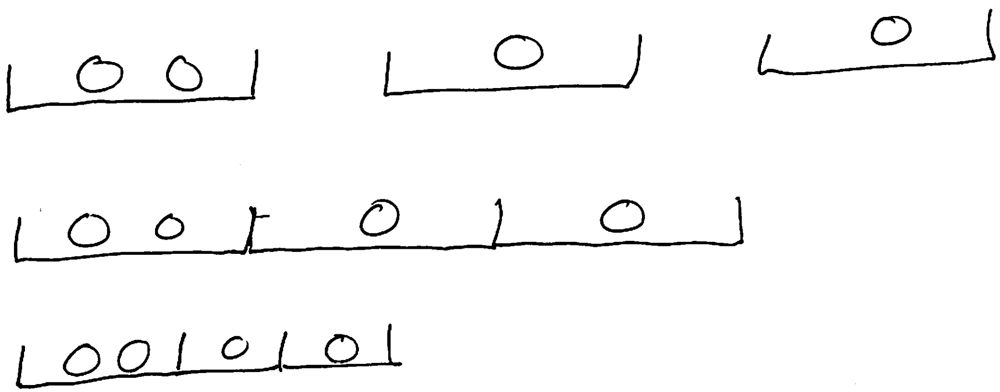
(humanities, tech, tech, tech)

(hum,)

(, hum,)

(, hum).





$\cancel{1}00101010\cancel{1}$

binary string of length

$3+4\cancel{+}-1 = 6$

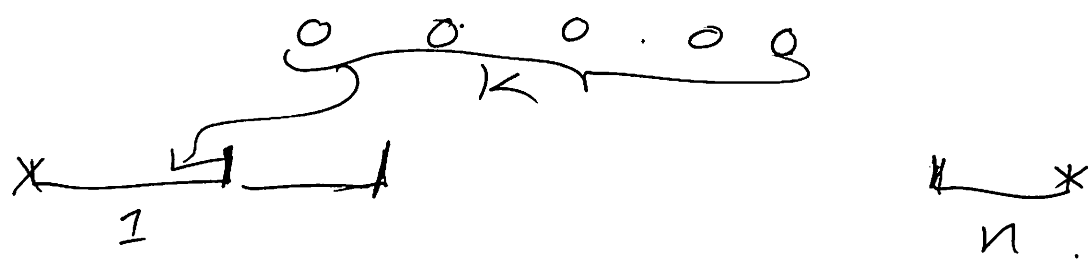
How many 1's $= \cancel{3+1}\ 3-1$

n choose K.

$\cancel{\binom{8}{4}} \binom{6}{2}$

$S = \{1, \ldots, n\}$.

Pick K elements.



$0\,0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0$

$n-1$ walls for boxes $=$ 1's.

K items. $=$ 0's.

$n+K-1$ length binary string with K 0's.

$$\binom{n+K-1}{K}$$