CS70 Fall 2013
Lecture 14

Sample Spaces, Events.

with/does $S = \{1, 2, \ldots, 6\}$. $k = 2$.

Probabilistic Experiment:

with/does

a) Toss a coin 4 times. Chance of exactly 2 Hs.
$S = \{H, T\}$ $k = 4$

$S = \{(1,1), (1,2), \ldots, (6,6)\}$
$n = 36$
$k = 1$.

b) Roll two dice. Chance of getting a pair.

$S = \{1, \ldots, 52\}$. $k = 5$

without/does c) Deal a poker hand. Chance of a flush.

$S = \{1, \ldots, 10\}$ $k = 20$ with/does.
d) Throw 20 balls in 10 bins at random.
Chance that bin 1 is empty.

$S = \{\text{flags}\}$.
e) Choose 8 flags at random from collection to fly on mast. Chance 3 consecutive flags have same color.

Probabilistic Experiment:

$S = \{1, 2, \dots, n\}$.

Pick (sample) sequence of k elements from S at random.

4 scenarios: with/without replacement.
order does/doesn't matter.

Toss coin 3 times: $S = \{H, T\}$.

omega
$\underline{\underline{\omega}}$ <u>outcome</u> — sample point. $\omega = HTH$

Sample space $= \Omega =$ set of all possible outcomes.

Experiment:
Picking an element of Ω at random.
Uniform distribution: each element of Ω is equally likely.

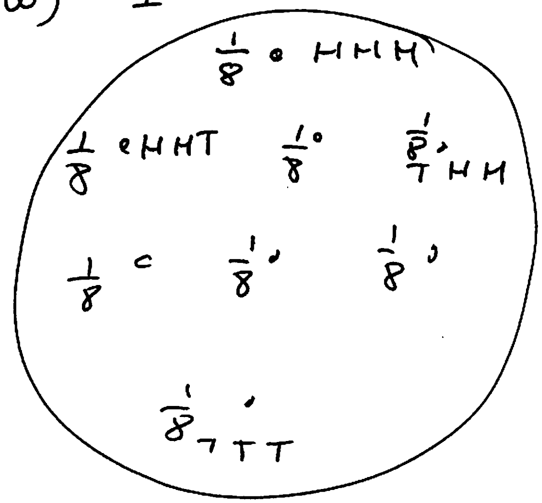
$\Omega =$ { HHH, HHT, HTH, THH, TTH, THT, HTT, TTT }

Sample Space Ω.

Assign a probability to each $\omega \in \Omega$

$$0 \le P(\omega) \le 1. \quad \sum_{\omega \in \Omega} P(\omega) = 1$$

<u>Uniform</u>: $\quad P(\omega) = \frac{1}{|\Omega|}$

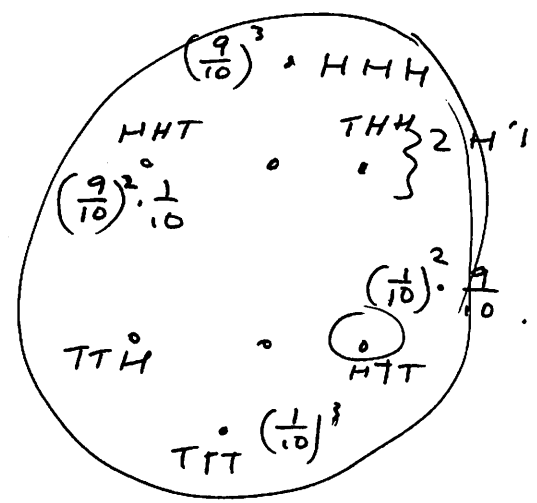


Biased coin:

$P[H] = \frac{9}{10}. \qquad P(T) = \frac{1}{10}.$

$P[HHH] = \left(\frac{9}{10}\right)^3 \qquad P(TTT) = \left(\frac{1}{10}\right)^3$

$P[HTH] = \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10}$



$P[H] = p \qquad P(T) = 1 - p.$

flip K times.

$$P[\underbrace{HH\cdots H}_{j}\ \underbrace{TT\cdots T}_{K-j}] = p^j (1-p)^{K-j}$$

Event:

$$\Omega = \{ HHH,\ HHT,\ HTH,\ THH,\ HTT,\ THT,\ TTH,\ TTT \}$$

$P[\text{exactly } 2\ H\text{'s}] = \dfrac{3}{8}$

$P[2^{nd} \text{ coin flip } H\text{s}] = \dfrac{4}{8}$

$P[\text{all 3 outcomes are the same}]$.

An event is a subset of Ω.

$A \subseteq \Omega$.

<u>Uniform</u>: $P(A) = \dfrac{|A|}{|\Omega|}$.

Non uniform: $P(A) = \sum_{\omega \in A} P[\omega]$.

$= \sum_{\omega \in A} \dfrac{1}{|\Omega|}$.

$= \dfrac{|A|}{|\Omega|}$

Order matters.

Roll two dice. $S = \{1, \cdots, 6\}$.

$k = 2$.

$P[\text{pair}]$?

Order matters: $|\Omega| = 36$.

$$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}.$$

$$P[A] = \frac{6}{36} = \frac{1}{6}.$$

Order doesn't matter: with replacement.

$S = \{1, 2, \ldots, 6\}$.

A - 6 possibilities. $\frac{30 \text{ others}}{2}$

$$|\Omega| = 6 + \frac{30}{2} = 21.$$

$$P[A] = \frac{6}{21} = \frac{2}{7}.$$

Poker hands: $P[\text{flush}]$? without replacement.

$$\frac{52!}{47!}$$ exactly $5!$ ways of dealing each hand of 5 cards.

$$\therefore \# \text{hands} = \frac{52!}{47!\,5!} = \binom{52}{5}.$$

$$|\Omega| = \binom{52}{5}.$$

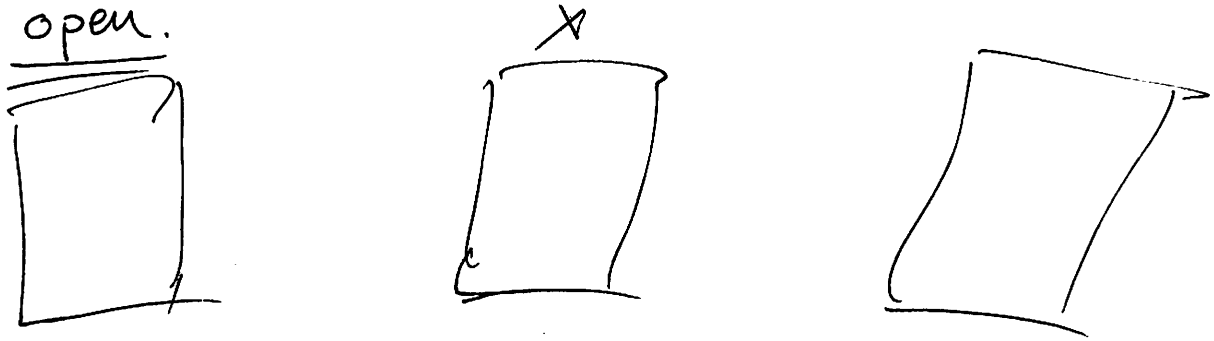
Flush: all cards same suit.

(Choose suit) then (choose 5 cards ~~from~~ of that suit).

$$\overset{\text{flush}}{\overset{\shortparallel}{|A|}} = 4 \times \binom{13}{5}$$

$$P[\text{flush}] = \frac{4\binom{13}{5}}{\binom{52}{5}} = \frac{4 \times \frac{13 \times 12 \times 11 \times 10 \times 9}{\cancel{5!}}}{\frac{52 \times 51 \times 50 \times 49 \times 48}{\cancel{5!}}} = \underline{\underline{.002}}$$

Montek Hall Paradox



1 Contestant chooses door.

2. Card opens one of other doors to reveal a goat.

3. Contestant is asked whether switch/stay.

I

Doesn't matter

50-50

II

Switch.

monty hall

September 10, 1990

Mr. Lawrence A. Denenberg
Harvard University Center for
Research in Computing Technology
Aiken Computation Laboratory, Room 102
Harvard University
Cambridge, MA 02138

Dear Larry:

In sending you my okay for the use of "The Monty Hall Paradox," I should like to ask you a question. You mention that in part (a), the player should switch doors even without additional compensation -- indeed the player should be willing to pay Monty up to $21,845 for the privilege of switching.

Now, I am not well versed in algorithms; but as I see it, it wouldn't make any difference after the player has selected Door A, and having been shown Door C - why should he then attempt to switch to Door B? The major prize could only be in one of the three doors. He has made his selection of one of the doors. He has been shown one of the doors that contains a "booby"; ergo, the major prize will be either in the one he selected (Door A) or the one that remains, Door B. Why would he be compelled to switch doors and even pay for the privilege? The chances of the major prize being behind Door A have not changed. He still has one of the two remaining doors. What makes Door B such an attraction? I would be pleased if you would write me, explaining this situation.

Best of luck with the book.

Sincerely,

Monty Hall

MH:ji