CS 70 Fall 2013
Lecture 15

Conditional Probability

Coin Flipping : 10 times Fair coin.

First 9 flips are H's.

$P[10^{th} \text{ flip } H] = \frac{1}{2}$.

independent

Deck of cards $B \leftrightarrow H$ $R \leftrightarrow T$.

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$P[1^{st} \text{ card is } B] = \frac{1}{2}$

$P[2^{nd} \text{ card is } B] = \frac{1}{2}$ — Assume we didn't look at 1^{st} card.

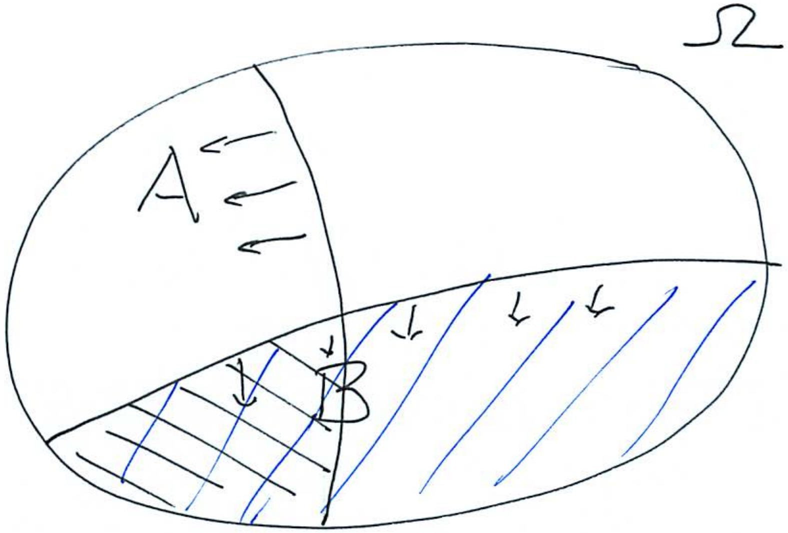
$P[2^{nd} \text{ card is } B \text{ if } 1^{st} \text{ card is } B] = ? = \frac{25}{51}$

$P[27^{th} \text{ card is } B \text{ if first 26 are all } B] = 0$

Hot Hand

$$P[\text{Basket} \mid \text{previous attempt Basket}] \approx P[\text{Basket}]$$

Ω = sample space



$$P[A] = \frac{|A|}{|\Omega|}$$

$$P[B] = \frac{|B|}{|\Omega|}.$$

P[A occurs given B occurs] = ?

$$P[A \mid B] = \frac{|A \cap B|}{|B|}$$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]}.$$

4 cards $\quad 2B \quad 2R$.

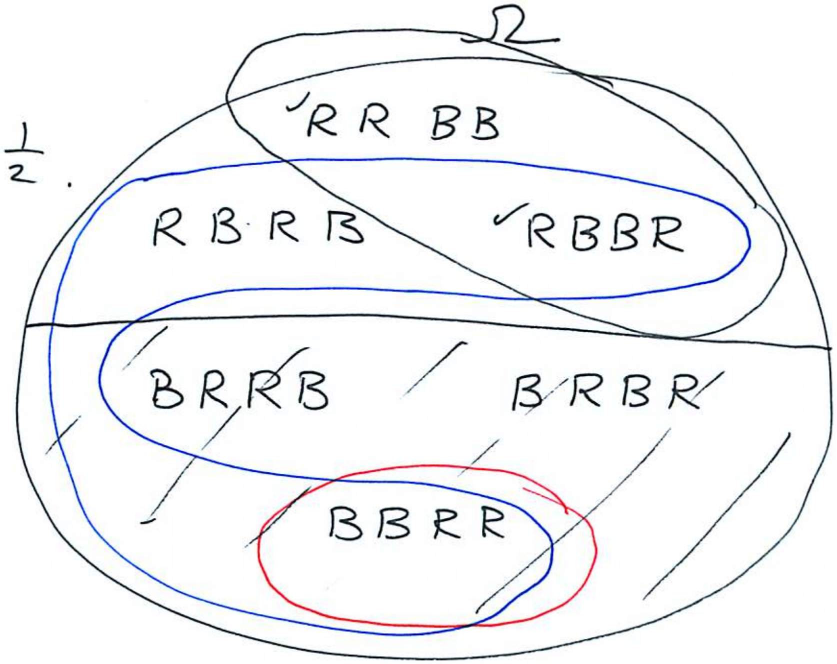
$P[\text{1st card } B] = \frac{3}{6} = \frac{1}{2}$.

$P[\text{2nd card } B] = \frac{3}{6} = \frac{1}{2}$

$P[\text{2nd card } B \,/\, \text{1st card } B)$

$= \frac{1}{3}$

$P[\text{3rd card } B \,/\, \text{1st } \cancel{\text{two}} \,\&\, \text{2nd } B] = 0$



$\binom{4}{2} = 6$

$P[\text{2nd card is } B \mid \text{1st is } R \,\&\, \text{3rd is } B] = \frac{1}{2}$

Independence:

A & B are independent events

$$P[A/B] = P[A]$$

$$\frac{P[A\cap B]}{P[B]} = P[A]$$

$$P[A\cap B] = P[A]\cdot P[B].$$

Biased coin $P[H] = p$ $P[T] = 1-p$.

Flip coin n times $P[\underbrace{HHHH\cdots H}_{k \text{ times}}\underbrace{T\cdots T}_{n-k \text{ times}}]$

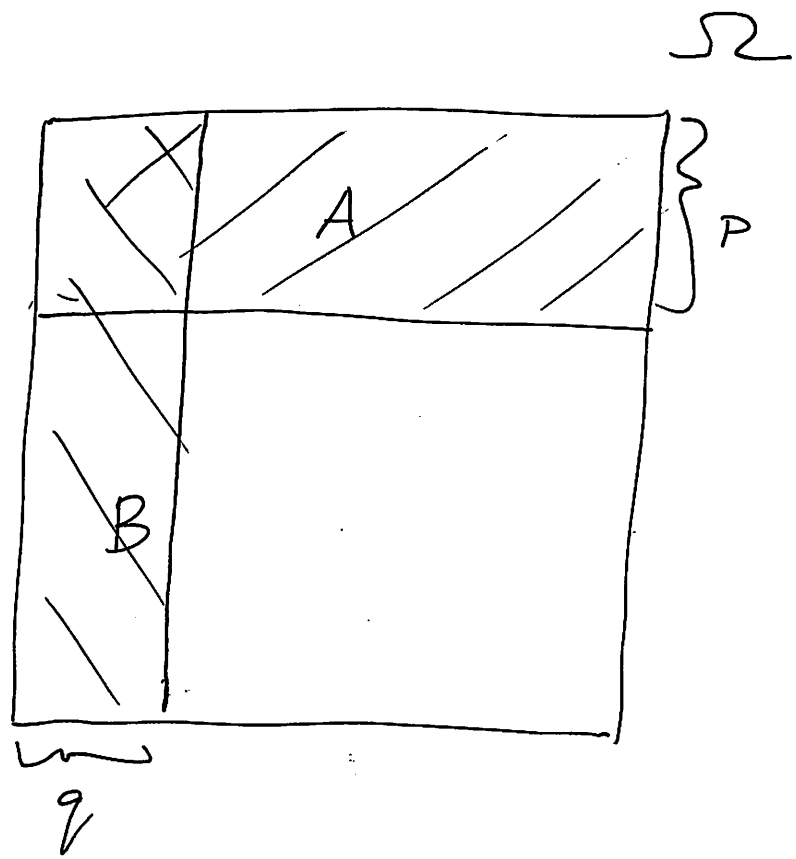
$$\| $$

$$p^k(1-p)^{n-k}.$$

$P[HH]$ $A =$ 1st toss H's. $P[A] = p$.

$B =$ 2nd toss H's. $P(B) = p$.

$$P[A\cap B) = P[A]\cdot P(B) = p^2.$$



$$\frac{|A|}{|\Omega|} = p$$

$$\frac{|B|}{|\Omega|} = q .$$

Bayes Rule

$$P[A|B] = \frac{P[A\cap B]}{P[B]} = \frac{P[B|A]\,P[A]}{P[B]}$$

A = Kidney cancer $\qquad P[A] = .001$

B = Microscopic Hematuria $\qquad P[B] = .05$

MH

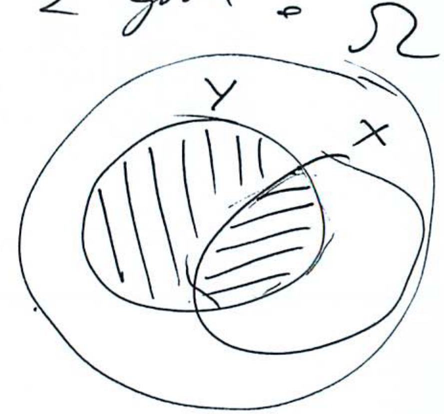
$$P[B|A] = 1$$

$$P[A|B] = \frac{P[B|A]\,P[A]}{P[B]} = \frac{1 \times .001}{.05}$$

$$= .02$$

each contains 2 coins.
one of them contains 2 Gold coins.
other contains 1 gold & 1 silver.

Choose a coin at random from A and it turns out to be gold.
What is the chance that A contained 2 gold?



X = A contains 2 gold.
Y = choose a gold coin from A.

$$P[X|Y] = \frac{P[Y|X]\,P[X]}{P[Y]}$$

$P[Y|X] = 1$ $P[X] = \frac{1}{2}$ $P[Y|\overline{X}] = \frac{1}{2}$ $P[\overline{X}] = \frac{1}{2}$.

Rule of total Prob: $P[Y \cap X] + P[Y \cap \overline{X}]$

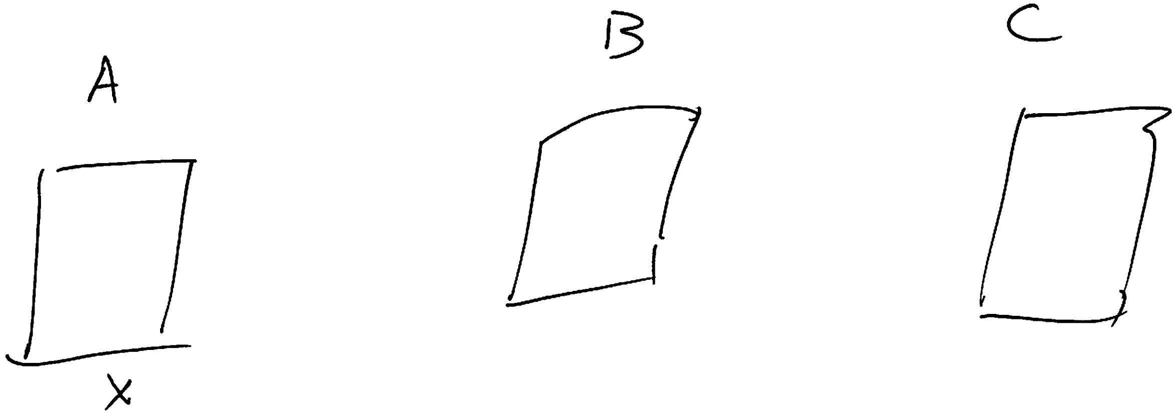
$$P[Y] = P[Y|X]\,P[X] + P[Y|\overline{X}]\,P[\overline{X}].$$

$$P[X|Y] = \frac{P[Y|X]\,P[X]}{P[Y|X]\,P[X] + P[Y|\bar{X}]\,P[\bar{X}]}.$$

$$= \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{\frac{1}{2}}{3/4}$$

$$= \frac{2}{3}.$$

Monte Hall Paradox



Contestant picks a door.

Host opens one of the remaining doors to reveal goat.

Contestant is allowed to switch.

$P[\text{contestant wins if stick}] = 1/3. \Rightarrow P[\text{win if switch}] = 2/3.$