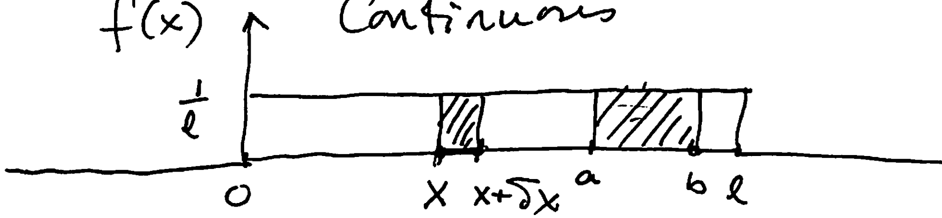
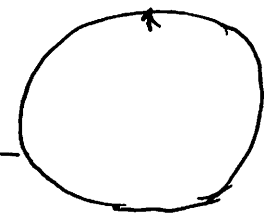
Continuous Probability

Continuous

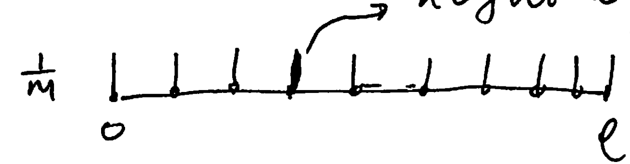




Discrete

$f(x)\,\delta x$ $=$

height $\approx \int_x^{x+\delta x} f(x)\,dx$



m parts. $\quad m \longrightarrow \infty$

$\cancel{P[X] = 0}$

$P[a \le X \le b] = \cancel{\frac{|a-b|}{\ell}}\ \frac{b-a}{\ell}.$

Probability density function.

$f(x)$

$$P[a \le x \le b] = \int_a^b f(x)\,dx.$$

$$E(X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^{\ell} x \frac{1}{\ell}\,dx = \frac{1}{\ell}\left.\frac{x^2}{2}\right|_0^{\ell} = \ell/2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\ell} x^2 \cdot \frac{1}{\ell}\,dx$$

$$= \frac{1}{\ell}\left.\frac{x^3}{3}\right|_0^{\ell} = \frac{\ell^2}{3}.$$

$$Var(x) = E(x^2] - (E[x])^2 = \frac{\ell^2}{3} - \left(\frac{\ell}{2}\right)^2 = \ell^2/12$$

$$P\left[X = \frac{j\ell}{m}\right] = \frac{1}{m}.$$

$j = 0, \ldots, m-1.$

$$E[X] = \sum_k k\,P[X=k]$$

$$= \ell/2.$$

$$Var(X) = E[X^2] - (E[X])^2$$

$$E[X^2] = \sum_k k^2 P[X=k]$$

$$= \sum_{j=0}^{m-1}\left(\frac{j\ell}{m}\right)^2 \cdot \frac{1}{m}.$$

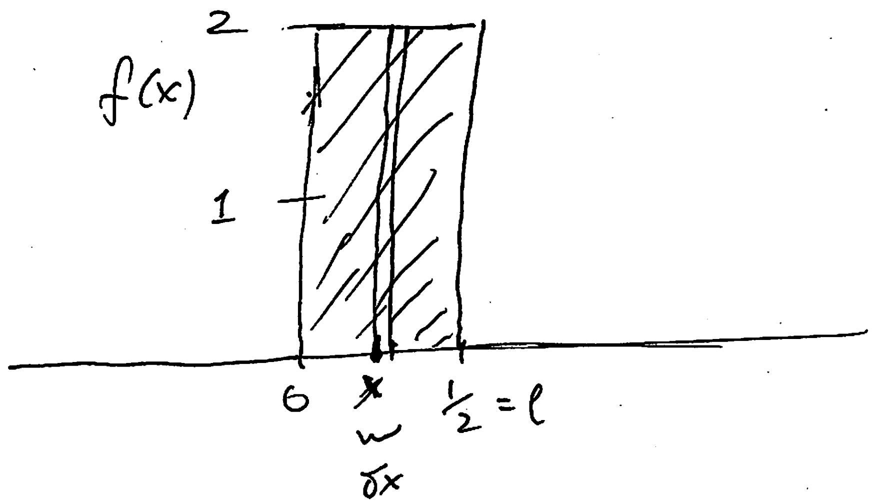
$$= \frac{\ell^2}{m^3}\sum_{j=0}^{m-1} j^2 \approx \frac{\ell^2}{m^3}\,\frac{m(m+1)(2m+1)}{6}$$

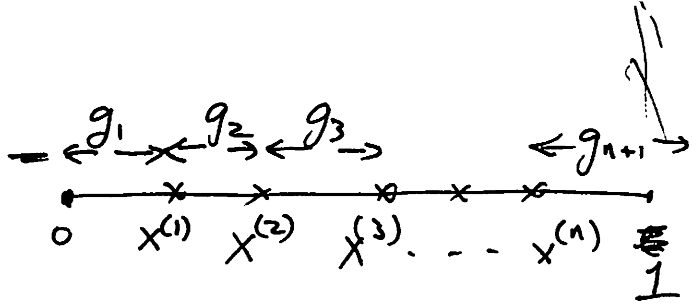
$$\approx \frac{\ell^2\, 2m^3}{m^3\, 6} = \frac{\ell^2}{3}.$$

$$Var(X) = \frac{\ell^2}{3} - \frac{\ell^2}{4} = \ell^2/12$$

$f(x) \geq 0$ $\underline{Q}$ $f(x) \leq 1$?

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$





n darts.

$$\cancel{P[X^{(1)} = a] =}$$

$$g(a)\,\delta = P[a \le X^{(1)} \le a+\delta] = n\frac{\delta}{1}(1-a)^{n-1}$$
$$= n\,\delta\,(1-a)^{n-1}$$

Density function $g(a)$

$$= n(1-a)^{n-1}$$

$$E[X^{(1)}] = \int_{-\infty}^{\infty} x\, g(x)\, dx$$

$$g_1 = X^{(1)}$$
$$g_2 = X^{(2)} - X^{(1)}$$
$$\vdots$$
$$g_n = X^{(n)} - X^{(n-1)}$$
$$g_{n+1} = 1 - X^{(n)}$$

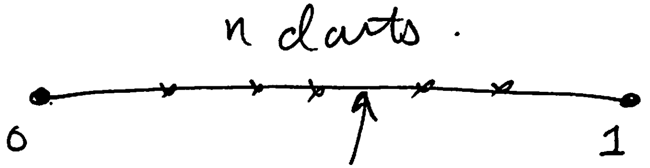
$$g_1 + g_2 + \cdots + g_{n+1} = 1.$$

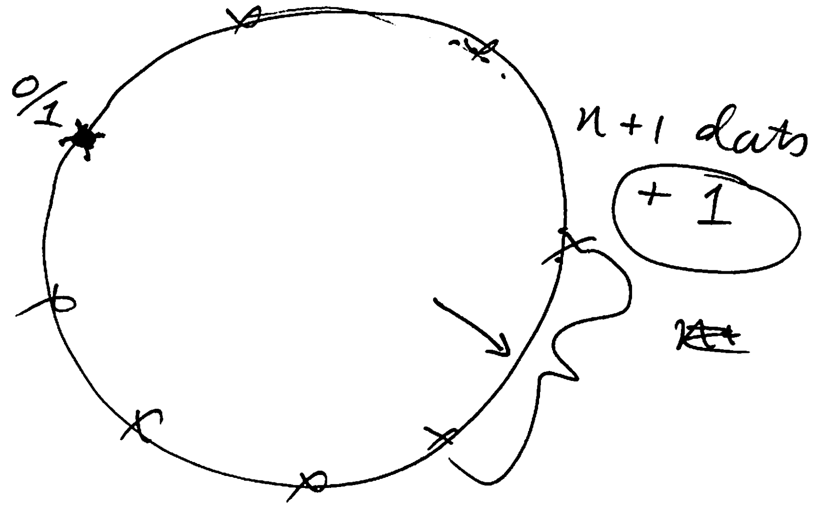
$$E(g_1) + E(g_2) + \cdots + E(g_{n+1}) = 1.$$

If $E(g_i) = E(g_j)\ \forall\, i, j$

then $n+1\ E(g_1) = 1.$

$$E(g_1) = \frac{1}{n+1}$$

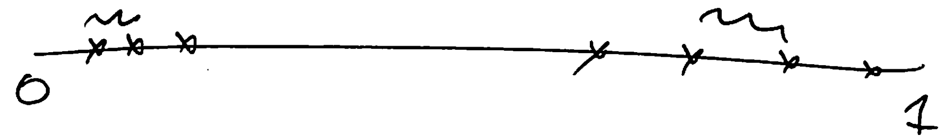




E(length of gap you point to) =

n+2 darts. 2 successive gaps.

$$\frac{1}{n+2} \times 2 = \frac{2}{n+2}.$$



Exponential Distribution:

λ = # H's per unit time
clicks

X = waiting time till first H's (click) etc.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda \geq 0 \\ 0 & \text{o/w} \end{cases}$$

density fnctn

$E[X] = 1/\lambda$ $\quad$ $Var(X) = 1/\lambda^2$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^{\infty} \lambda e^{-\lambda x}\,dx$$

$$= \lambda\left(-\frac{1}{\lambda}\right) e^{-\lambda x} \Big|_0^{\infty} = 1.$$

$$E(X) = \int_0^{\infty} x f(x)\,dx = \int_0^{\infty} x \lambda e^{-\lambda x}\,dx$$

Flip a coin n times / ~~unit~~ min $\qquad P[H] = \frac{\lambda}{n}$.

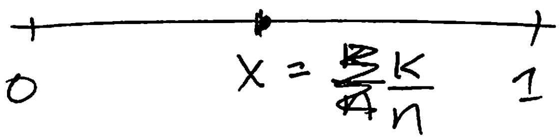
$\cancel{P\left[a \le X \le a + \varepsilon\right]}$ $\quad \varepsilon = \frac{1}{n}$

$$P[W_1 = k] = \left(1 - \frac{\lambda}{n}\right)^{k-1} \frac{\lambda}{n}.$$

~~P[H's an~~

$$f(x) \cdot \frac{1}{n} \approx P\left[X = \frac{k}{n}\right] = \left(1 - \frac{\lambda}{n}\right)^{k} \frac{\lambda}{n}.$$

$$= \left(\left(1 - \frac{\lambda}{n}\right)^{\frac{n}{\lambda}}\right)^{\frac{\lambda}{n}k} \frac{\lambda}{n}$$



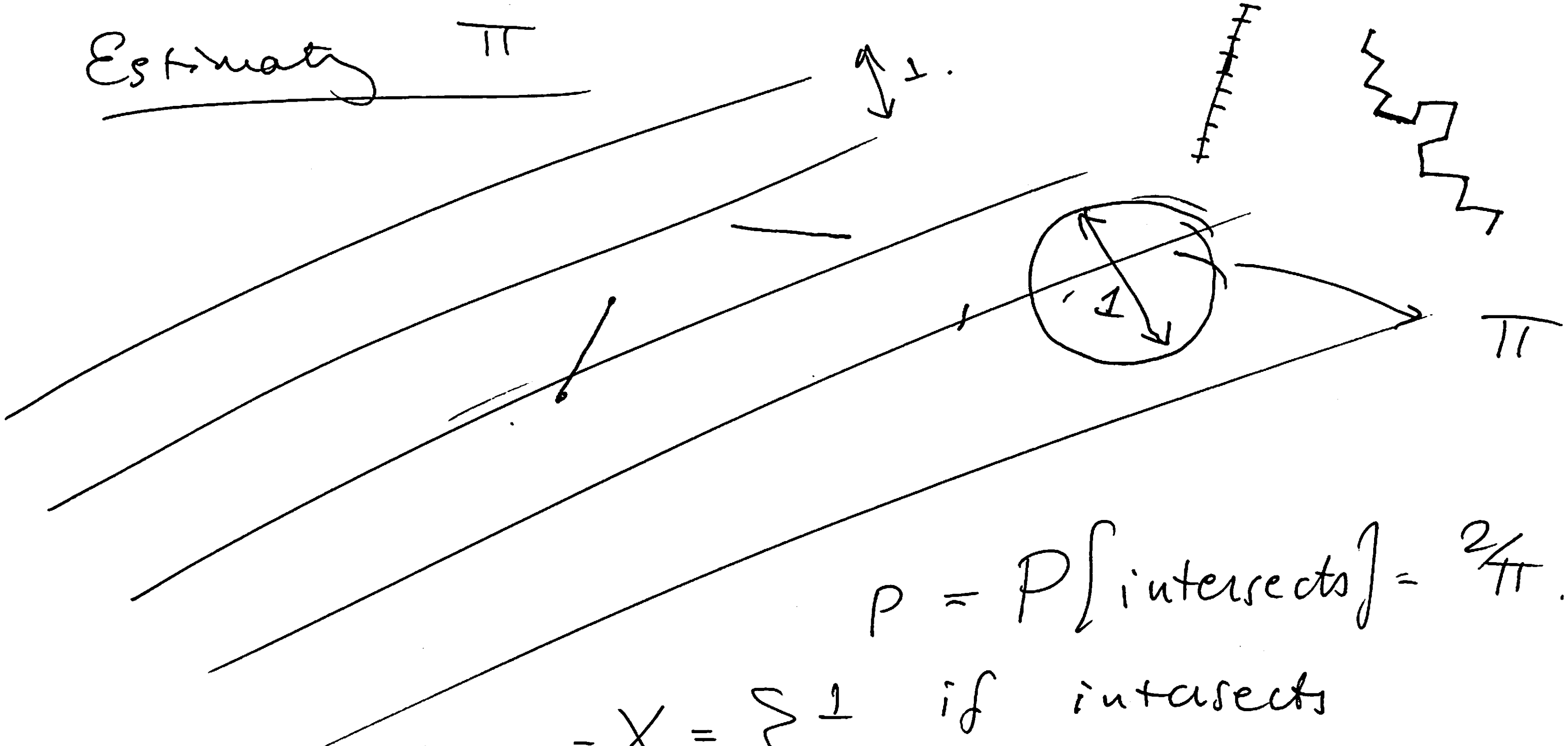
$$= e^{-\frac{\lambda k}{n}} \cdot \frac{\lambda}{n}$$

$$(1-y)^{\frac{1}{y}} \to \frac{1}{e}.$$

$$= e^{-\lambda x} \cdot \frac{\lambda}{n}.$$

$$f(x) = e^{-\lambda x} \cdot \lambda.$$

Estimating π



$$p = P[\text{intersects}] = 2/\pi.$$

$$\#\text{ intersects} = X = \begin{cases} 1 & \text{if intersects} \\ 0 & \text{o/w.} \end{cases}$$

$$E[X] = p.$$

$$Y\ \cancel{Z} = X_1 + \cdots + X_n \qquad X_i = \begin{cases} 1 & \text{if segment } i \text{ intersects line} \\ 0 & \text{o/w.} \end{cases}$$

$$E[\cancel{Y}] = 2 = \pi * E[X]$$

$$E[X] = 2/\pi.$$