Self Reference, Diagonalization and Uncomputability.

Induction & Recursion

Analysis of algorithms

This statement is false

Russell's Paradox:

$S = \text{all men in village who don't shave themselves}$

Barber $\in S$?
**Axioms**

**Peano Axioms (Natural Numbers):**

1. $0 \in \mathbb{N}$.
2. $x \in \mathbb{N} \Rightarrow x+1 \in \mathbb{N}$.
3. $x+1 = y+1 \Rightarrow x = y$.
4. $\exists x : x+1 = 0$.

5. **Induction Axiom:**
   
   $S \subseteq \mathbb{N}$
   
   $0 \in S$ and if $x \in S \Rightarrow x+1 \in S$
   
   then $S = \mathbb{N}$.

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**Proofs**

\[
A \Rightarrow B \quad \exists \quad \text{infer} \quad B
\]

All men are mortal $\exists \Rightarrow$ Socrates is mortal.

$\forall n \geq 2 \quad \forall x, y, z \in \mathbb{N} \setminus \{0\} : x^n + y^n = z^n$
Gödel: Cannot write down a set of axioms.
Prove that all true statements follow from axioms via proofs.

Consistency: Cannot prove consistency
If you prove consistent
then it is inconsistent.
Halting Problem:

Assume $\exists \text{Turing} (P)$

$\text{Turing} (Turing)$ halts?

Contradiction.
Program: $\rightarrow$ data $\rightarrow$

$P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow P_4 \rightarrow \cdots \rightarrow P_n \rightarrow \cdots$

$P_1 \rightarrow H \rightarrow L \rightarrow H \rightarrow H \rightarrow \cdots$

$P_2 \rightarrow L \rightarrow L \rightarrow H \rightarrow H \rightarrow \cdots$

$P_3 \rightarrow \cdots$

$P_4 \rightarrow \cdots$

$\vdots$

$P_n \rightarrow \cdots$

Turing $\rightarrow P_n \\
\vdots$

Turing$(P)$ does the opposite of $P(P)$.

$\therefore$ Turing does not occur on the list of all programs.

$\Rightarrow$ Turing does not exist
Does program $P$ ever execute a certain line of code?

How to use a program $T$: Test Execute as a subroutine in a program to solve the Halting problem.

$$(P, I) \rightarrow (\text{Test Execute}) \rightarrow \text{yes/} \rightarrow \text{no.}$$

Assume for contradiction that Test Execute exists. Show how to use it to implement Test Halt.

Contradiction: $T$: Test Execute does not exist.
Virus

```
(quote "compute") = (compute "compute")

(quote "quine") = (quote "quine")
```
Induction vs Recursion
Analysis of Algorithms

* Modular arithmetic
  Probability

* Model: Stable Marriage
  Error correct codes
  Load Balancing
  Hashing

* Problem solving