Introduction to Graphs

Note: you aren’t expected to complete even all of the non-challenge problems. Extra problems are included to help with practice.

1. Give the necessary and sufficient conditions for an undirected graph to have an Eulerian walk.

2. A Hamiltonian path is a path that visits each vertex exactly once. A tournament graph is a directed graph such that for all vertices \( u, v \) in the graph, either \((u, v) \in E\) or \((v, u) \in E\). Show that a tournament graph has a Hamiltonian path.

3. Challenge problem: In lecture, you learned that an undirected graph \( G = (V, E) \) has an Eulerian tour if and only if the graph is connected (except for isolated vertices) and even degree. Prove the following alternate characterization of Eulerian graphs: A connected graph \( G \) has an Eulerian tour if and only if its edge set can be decomposed into disjoint cycles (two cycles are disjoint if they share no edges). Hint: try using induction on the number of edges in the graph.