CS 70 Discrete Mathematics and Probability Theory
Fall 2015 Jean Walrand Discussion 11B

1. **(Sanity Check!)** Define a random variable X to be the result of rolling two standard dice and summing the results.

   (a) Directly compute the expectation of X.

   (b) Compute the expectation of X using linearity of expectation.

2. **(True or False?)** If X, Y are random variables, and $E[XY] = E[X]E[Y]$, then X and Y are independent.

3. **(Geometric Distribution)** Coco is trying to collect four coupons, C_1, C_2, C_3, C_4. She does not know which coupon(s) she will get before paying. There are two stores selling coupons:

   - In Store A, she pays 3 dollars to get a coupon, and the probability of getting each coupon is always $\frac{1}{4}$.
   - In Store B, C_1 and C_2 are always packaged together, and C_3 and C_4 are also always packaged together. She pays 8 dollars to get one package (with two coupons), and the probability of getting each pair (either (C_1, C_2) or (C_3, C_4)) is always $\frac{1}{2}$.

   Which store should she go, or it does not matter?

4. **(Conditional Expectation)** Suppose the number of children in a family is a random variable X with mean μ, and given $X = n$ for $n \geq 1$, each of the n children in the family is a girl with probability p and a boy with probability $1 - p$. What is the expected number of girls in a family?

5. **(James Bond)** James Bond is playing a game that involves repeatedly rolling a fair standard 6-sided die.

   (a) What is the expected number of rolls until he gets a 5?

   (b) What is the expected number of rolls until the last two rolls sum to 7?