CS 70 Discrete Mathematics and Probability Theory
Fall 2015 Jean Walrand Discussion 12B

1. **(Sanity Check!)** Derive Chebyshev's inequality using Markov's inequality for random variable X.

2. **(Balls and Bins Again)** For this problem we toss m balls into n bins.

   (a) What is the expected number of collisions?

   (b) Now, let's define X to be the number of collisions. At what threshold of collisions c can we ensure that the probability of having more than c collisions is less than $1/2n$?

3. **(Coupon Collector)** There are n baseball cards Jonny is trying to collect! Each day, Jonny buys a cereal box and dumps out all the cereal, searching for the baseball card. Each box contains each of the n cards with equal probability.

   (a) What's the expected number of days until Jonny gets one unique card? Two unique cards?

   (b) Develop a formula for the expected number of days until Jonny gets k unique cards. On average, how many days will it take Jonny to collect all the baseball cards? Approximate your sum with an integral.

(c) Jonny and his sister, Jill, are working together to collect the cards. Each day, they both buy cereal boxes. On average, how many days will it take to collect k unique baseball cards? How many days until they collect them all? Be exact.

4. **(Coin flips)**

(a) Suppose we flip a fair coin n times and we wish to understand the probability that we get at least $3n/4$ heads. Use Markov's inequality to come up with an upper bound for this probability.

(b) Use Markov's inequality to come up with a similar upper bound on the probability that the number of heads is at least n.

(c) Find the true probability that there are at least n heads in a sequence of n fair coin flips. Is the bound you derived in the previous part tight?

5. **(More coin flips)**

(a) Suppose we flip a biased coin 100 times and X is the number of heads we get. We know that $\mathrm{Var[X]} = 16$. What are the possible values for the expected value of X?

(b) Now suppose $\mathbf{E}[X] = 20$. Use Chebyshev's inequality to derive an upper bound on $\Pr[X \geq 40]$.