CS 70 Discrete Mathematics and Probability Theory
Fall 2015 Jean Walrand 15A Discussion

1. 007 Escapes!

   James Bond is imprisoned in a cell from which there are three possible ways to escape: an air-conditioning duct, a sewer pipe and the door (which is unlocked). The air-conditioning duct leads him on a two-hour trip whereupon he falls through a trap door onto his head, much to the amusement of his captors. The sewer pipe is similar but takes five hours to traverse. Each fall produces amnesia and he is returned to the cell immediately after each fall. Assume that he always immediately chooses one of the three exits from the cell with probability $\frac{1}{3}$. On average, how long does it take before he opens the unlocked door and escapes?

2. Family Planning

   Mr and Mrs Brown decide to continue having children until they either have their first girl or until they have five children. Assume that each child is equally likely to be a boy or a girl, independent of all other children, and that there are no multiple births. Let B and G denote the numbers of boys and girls respectively that the Browns have. Let C be the total number of children they have.

   (a) Write down the sample space together with the probability of each sample point.

   (b) Write down the distributions of the random variables B, G and C.

   (c) Write down the joint distribution of G and C.

   (d) Write down the conditional distributions of C given $G = i$ for all possible values i that G can take on.

3. Continuous Probability
   Let X be a continuous r.v. with probability density function of the form :

$$f(x) = ce^{\lambda|x|}, \qquad -\infty < x < +\infty$$

   (a) Can λ take on any arbitrary value? If so, explain. If not, specify the possible values λ can take on.
   (b) Sketch the probability density function.
   (c) Given a particular λ, can c take on any arbitrary value? If so, explain. If not, specify the possible values that c can take on (in terms of λ).
   (d) Evaluate $\mathbf{P}[X = 2]$.
   (e) Evaluate the mean of X.