Today

Review for Midterm.
First there was logic...
A statement is a true or false.
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A statement is a true or false.

Statements?
First there was logic...
A statement is a true or false.

Statements?
3 = 4 − 1 ?
First there was logic...
A statement is a true or false.

Statements?
  $3 = 4 - 1$ ? Statement!
First there was logic...

A statement is a true or false.

Statements?

$3 = 4 - 1$ ? Statement!

$3 = 5$ ?
First there was logic...

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Statements?

3 = 4 − 1? Statement!
3 = 5? Statement!
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Statements?

3 = 4 − 1 ? Statement!
3 = 5 ? Statement!
3 ?
First there was logic...

A statement is a true or false.

Statements?

\[ 3 = 4 - 1 \ ? \] Statement!
\[ 3 = 5 \ ? \] Statement!
\[ 3 \ ? \] Not a statement!

Predicate: Statement with free variable(s).

Example:
\[ x = 3 \]
Given a value for \( x \), becomes a statement.

Predicate?

\[ n > 3 \ ? \] Predicate:
\[ P(n) \]

Quantifiers:

\[ \forall x \in \mathbb{R}, x^2 \geq x \]

\[ \forall x \in \mathbb{R} \exists y \in \mathbb{R}, y > x \]
First there was logic...

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Statements?

$3 = 4 - 1$? Statement!
$3 = 5$? Statement!
$3$? Not a statement!
$n = 3$?
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Statements?

3 = 4 – 1 ? Statement!
3 = 5 ? Statement!
3 ? Not a statement!

n = 3 ? Not a statement...
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Statements?
\[ 3 = 4 - 1 \ ? \] Statement!
\[ 3 = 5 \ ? \] Statement!
\[ 3 \ ? \] Not a statement!
\[ n = 3 \ ? \] Not a statement...but a predicate.
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Example: x = 3   Given a value for x, becomes a statement.
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Example: \( x = 3 \) Given a value for \( x \), becomes a statement.

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  3 = 4 − 1 ? Statement!
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Predicate: Statement with free variable(s).
  Example: x = 3  Given a value for x, becomes a statement.
Predicate?
  n > 3 ?
First there was logic...

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Statements?

- $3 = 4 - 1$ ? Statement!
- $3 = 5$ ? Statement!
- $3$ ? Not a statement!
- $n = 3$ ? Not a statement...but a predicate.

Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for $x$, becomes a statement.

Predicate?

- $n > 3$ ? Predicate: $P(n)$!
First there was logic...

A statement is a true or false.

Statements?

$3 = 4 - 1$ ? Statement!

$3 = 5$ ? Statement!

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Predicate: Statement with free variable(s).

Example: $x = 3$  
Given a value for $x$, becomes a statement.

Predicate?

$n > 3$ ? Predicate: $P(n)$!

$x = y$?
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Predicate: Statement with free variable(s).

Example: \( x = 3 \)  
Given a value for \( x \), becomes a statement.

Predicate?

\[ n > 3 \ ? \] Predicate: \( P(n) \! \)

\[ x = y \? \] Predicate: \( P(x, y) \! \)
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Predicate: Statement with free variable(s).
Example: x = 3 Given a value for x, becomes a statement.

Predicate?
n > 3 ? Predicate: P(n)!
x = y? Predicate: P(x, y)!
x + y?
First there was logic...

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Statements?

\(3 = 4 - 1\) ? Statement!
\(3 = 5\) ? Statement!
\(3\) ? Not a statement!
\(n = 3\) ? Not a statement... but a predicate.

Predicate: Statement with free variable(s).
Example: \(x = 3\) Given a value for \(x\), becomes a statement.

Predicate?

\(n > 3\) ? Predicate: \(P(n)\)!
\(x = y\)? Predicate: \(P(x, y)\)!
\(x + y\) ? No.
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Predicate: Statement with free variable(s).

Example: x = 3 Given a value for x, becomes a statement.

Predicate?

n > 3 ? Predicate: P(n)!
x = y? Predicate: P(x, y)!
x + y? No. An expression, not a statement.
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Quantifiers:
(∀x) P(x).
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Example: x = 3 Given a value for x, becomes a statement.

Predicate?
n > 3 ? Predicate: P(n)!
x = y? Predicate: P(x, y)!
x + y? No. An expression, not a statement.

Quantifiers:
(∀x) P(x).
For every x, P(x) is true.
First there was logic...

A statement is a true or false.

Statements?
   \[3 = 4 - 1\] ? Statement!
   \[3 = 5\] ? Statement!
   \[3\] ? Not a statement!
   \[n = 3\] ? Not a statement...but a predicate.

Predicate: Statement with free variable(s).
   Example: \[x = 3\] Given a value for \(x\), becomes a statement.

Predicate?
   \[n > 3\] ? Predicate: \(P(n)\)!
   \[x = y\] ? Predicate: \(P(x, y)\)!
   \[x + y\] ? No. An expression, not a statement.

Quantifiers:
   \((\forall x)\ P(x)\).
      For every \(x\), \(P(x)\) is true.
   \((\exists x)\ P(x)\).
First there was logic...  
A statement is a true or false.

Statements?  
  \[ 3 = 4 - 1 ? \text{ Statement!} \]  
  \[ 3 = 5 ? \text{ Statement!} \]  
  \[ 3 ? \text{ Not a statement!} \]  
  \[ n = 3 ? \text{ Not a statement...but a predicate.} \]

Predicate: Statement with free variable(s).  
  Example: \( x = 3 \)  
  Given a value for \( x \), becomes a statement.

Predicate?  
  \[ n > 3 ? \text{ Predicate: } P(n)! \]  
  \( x = y? \text{ Predicate: } P(x, y)! \)  
  \( x + y? \text{ No. An expression, not a statement.} \)

Quantifiers:  
  \( (\forall x) \ P(x). \)  
  For every \( x \), \( P(x) \) is true.  
  \( (\exists x) \ P(x). \)  
  There exists an \( x \), where \( P(x) \) is true.
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n > 3 ? Predicate: \( P(n) \)!
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Quantifiers:

(\( \forall x \)) \( P(x) \).
For every \( x \), \( P(x) \) is true.

(\( \exists x \)) \( P(x) \).
There exists an \( x \), where \( P(x) \) is true.

(\( \forall n \in N \)), \( n^2 \geq n \).
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n > 3 ? Predicate: \( P(n) \)!
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(\( \forall x \) \( P(x) \)).
For every \( x \), \( P(x) \) is true.
(\( \exists x \) \( P(x) \)).
There exists an \( x \), where \( P(x) \) is true.

(\( \forall n \in N \), \( n^2 \geq n \).
(\( \forall x \in R \)(\( \exists y \in R \))\( y > x \).
First there was logic...
A statement is a true or false.

Statements?
\[3 = 4 - 1\ ? \text{ Statement!}\]
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\[x + y\? \text{ No. An expression, not a statement.}\]

Quantifiers:
\[\forall x \ P(x)\].
For every \(x\), \(P(x)\) is true.

\[\exists x \ P(x)\].
There exists an \(x\), where \(P(x)\) is true.

\[(\forall n \in N), n^2 \geq n.\]
\[(\forall x \in R)(\exists y \in R)y > x.]
Connecting Statements

$A \land B, A \lor B, \neg A.$
Connecting Statements

\[ A \land B, \ A \lor B, \ \neg A. \]

You got this!
Connecting Statements

\[ A \land B, \ A \lor B, \ \neg A. \]

You got this!

Propositional Expressions and Logical Equivalence
Connecting Statements

\[ A \land B, \ A \lor B, \ \neg A. \]

You got this!

Propositional Expressions and Logical Equivalence

\[ (A \implies B) \equiv (\neg A \lor B) \]
Connecting Statements

$A \land B, A \lor B, \neg A.$

You got this!

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \lor B)$
$(\neg (A \lor B)) \equiv (\neg A \land \neg B)$
Connecting Statements

\[ A \land B, \ A \lor B, \ \neg A. \]

You got this!

Propositional Expressions and Logical Equivalence

\[
(A \implies B) \equiv (\neg A \lor B) \\
(\neg (A \lor B)) \equiv (\neg A \land \neg B)
\]
Connecting Statements

$A \land B, A \lor B, \neg A.$

You got this!

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \lor B)$

$(\neg (A \lor B)) \equiv (\neg A \land \neg B)$

Proofs: truth table or manipulation of known formulas.
Connecting Statements

\[ A \land B, \ A \lor B, \ \neg A. \]

You got this!

Propositional Expressions and Logical Equivalence

\[(A \implies B) \equiv (\neg A \lor B)\]
\[\neg (A \lor B) \equiv (\neg A \land \neg B)\]

Proofs: truth table or manipulation of known formulas.

\((\forall x)(P(x) \land Q(x)) \equiv (\forall x)P(x) \land (\forall x)Q(x)\)
..and then proofs...

Direct: $P \implies Q$

Example:
- $a$ is even $\implies a^2$ is even.

Approach: What is even?

- $a = 2k$
- $a^2 = 4k^2$

What is even?

- $a^2 = 2(2k^2)$

Integers closed under multiplication!

$a^2$ is even.

Contrapositive:

$P \implies Q$ or $\neg Q = \implies \neg P$.

Example:
- $a^2$ is odd $\implies a$ is odd.

Contrapositive:
- $a$ is even $\implies a^2$ is even.

Contradiction:

$P \neg P = \implies false$
- $\neg P = \implies R \land \neg R$

Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes does not exist.

Example: rogue couple does not exist.
..and then proofs...

Direct: \( P \implies Q \)

Example: \( a \) is even \( \implies a^2 \) is even.
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Direct: $P \implies Q$

Example: $a$ is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

Integers closed under multiplication!

Contrapositive: $P \implies Q$ or $\neg Q = \implies \neg P$.

Example: $a^2$ is odd $\implies a$ is odd.

Contrapositive: $a$ is even $\implies a^2$ is even.

Contradiction: $P \neg P = \implies \text{false}$

$\neg P = \implies R \land \neg R$

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$a^2 = 4k^2$. 
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$a^2 = 4k^2$.

What is even?
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Example: $a$ is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

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What is even?

$a^2 = 2(2k^2)$
..and then proofs...

Direct: \( P \implies Q \)

Example: \( a \) is even \( \implies a^2 \) is even.

Approach: What is even? \( a = 2k \)

\[ a^2 = 4k^2. \]

What is even?

\[ a^2 = 2(2k^2) \]

Integers closed under multiplication!

Contrapositive:

\( P \implies \neg Q \) or \( \neg Q \implies \neg P \).

Example: \( a^2 \) is odd \( \implies a \) is odd.

Contrapositive: \( a \) is even \( \implies a^2 \) is even.

Contradiction:

\( P \neg P \implies \text{false} \)

\( \neg P \implies R \wedge \neg R \)

Useful for prove something does not exist:

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\[ a^2 = 2(2k^2) \]

Integers closed under multiplication!

$a^2$ is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: $a^2$ is odd $\implies a$ is odd.
..and then proofs...

Direct: \( P \implies Q \)

Example: \( a \text{ is even} \implies a^2 \text{ is even.} \)

Approach: What is even? \( a = 2k \)
\[ a^2 = 4k^2. \]

What is even?
\[ a^2 = 2(2k^2) \]

Integers closed under multiplication!
\( a^2 \text{ is even.} \)

Contrapositive: \( P \implies Q \text{ or } \neg Q \implies \neg P. \)

Example: \( a^2 \text{ is odd} \implies a \text{ is odd.} \)

Contrapositive: \( a \text{ is even} \implies a^2 \text{ is even.} \)
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Direct: $P \implies Q$
- Example: $a$ is even $\implies a^2$ is even.
  - Approach: What is even? $a = 2k$
    $a^2 = 4k^2$.
  - What is even?
    $a^2 = 2(2k^2)$
  - Integers closed under multiplication!
    $a^2$ is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.
- Example: $a^2$ is odd $\implies a$ is odd.
  - Contrapositive: $a$ is even $\implies a^2$ is even.

Contradiction: $P$
..and then proofs...

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Example: \( a \) is even \( \implies \) \( a^2 \) is even.
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Example: \( a^2 \) is odd \( \implies \) \( a \) is odd.
Contrapositive: \( a \) is even \( \implies \) \( a^2 \) is even.

Contradiction: \( P \)
\( \neg P \implies \text{false} \)
..and then proofs...

Direct: \( P \implies Q \)
- Example: \( a \) is even \( \implies a^2 \) is even.
  - Approach: What is even? \( a = 2k \)
    \( a^2 = 4k^2 \).
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- Example: \( a^2 \) is odd \( \implies a \) is odd.
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Contradiction: \( P \)
- \( \neg P \implies \text{false} \)
- \( \neg P \implies R \land \neg R \)
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   \( \neg P \implies \text{false} \)
   \( \neg P \implies R \land \neg R \)
Useful for prove something does not exist:
..and then proofs...

Direct: $P \implies Q$

Example: $a$ is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2$.

What is even?

$a^2 = 2(2k^2)$

Integers closed under multiplication!

$a^2$ is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: $a^2$ is odd $\implies a$ is odd.

Contrapositive: $a$ is even $\implies a^2$ is even.

Contradiction: $P$

$\neg P \implies \text{false}$

$\neg P \implies R \land \neg R$

Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$
..and then proofs...

Direct: \( P \implies Q \)
- Example: \( a \) is even \( \implies a^2 \) is even.
  - Approach: What is even? \( a = 2k \)
    \[ a^2 = 4k^2. \]
  - What is even?
    \[ a^2 = 2(2k^2) \]
  - Integers closed under multiplication!
    \( a^2 \) is even.

Contrapositive: \( P \implies Q \) or \( \neg Q \implies \neg P \).
- Example: \( a^2 \) is odd \( \implies a \) is odd.
  - Contrapositive: \( a \) is even \( \implies a^2 \) is even.

Contradiction: \( P \)
- \( \neg P \implies \text{false} \)
- \( \neg P \implies R \land \neg R \)

Useful for prove something does not exist:
- Example: rational representation of \( \sqrt{2} \) does not exist.
..and then proofs...

Direct: $P \implies Q$
- Example: $a$ is even $\implies a^2$ is even.
  - Approach: What is even? $a = 2k$
  - $a^2 = 4k^2$.
  - What is even?
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- Example: $a^2$ is odd $\implies a$ is odd.
  - Contrapositive: $a$ is even $\implies a^2$ is even.

Contradiction: $P$
- $\neg P \implies \textbf{false}$
- $\neg P \implies R \land \neg R$

Useful for prove something does not exist:
- Example: rational representation of $\sqrt{2}$ does not exist.
- Example: finite set of primes
..and then proofs...

Direct: \( P \implies Q \)

Example: \( a \text{ is even} \implies a^2 \text{ is even.} \)

Approach: What is even? \( a = 2k \)
\[ a^2 = 4k^2. \]
What is even?
\[ a^2 = 2(2k^2) \]
Integers closed under multiplication!
\( a^2 \text{ is even.} \)

Contrapositive: \( P \implies Q \) or \( \neg Q \implies \neg P. \)

Example: \( a^2 \text{ is odd} \implies a \text{ is odd.} \)
Contrapositive: \( a \text{ is even} \implies a^2 \text{ is even.} \)

Contradiction: \( P \)
\[ \neg P \implies \text{false} \]
\[ \neg P \implies R \land \neg R \]

Useful for prove something does not exist:
Example: rational representation of \( \sqrt{2} \) does not exist.
Example: finite set of primes does not exist.
..and then proofs...

Direct: $P \implies Q$

Example: $a$ is even $\implies a^2$ is even.

Approach: What is even? $a = 2k$

$a^2 = 4k^2$.

What is even?

$a^2 = 2(2k^2)$

Integers closed under multiplication!

$a^2$ is even.

Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

Example: $a^2$ is odd $\implies a$ is odd.

Contrapositive: $a$ is even $\implies a^2$ is even.

Contradiction: $P$

$\neg P \implies \text{false}$

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Integers closed under multiplication!

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes does not exist. Example: rogue couple does not exist.
...jumping forward..

Contradiction in induction:
...jumping forward..

Contradiction in induction:
contradict place where induction step doesn’t hold.
...jumping forward..

Contradiction in induction:
   contradict place where induction step doesn’t hold.

Well Ordering Principle.
...jumping forward..

Contradiction in induction:
   contradict place where induction step doesn’t hold.

Well Ordering Principle.
Stable Marriage:
...jumping forward..

Contradiction in induction:
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Stable Marriage:
first day where women does not improve.
...jumping forward..

Contradiction in induction:
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Stable Marriage:
first day where women does not improve.
first day where any man rejected by optimal women.
...jumping forward..

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Stable Marriage:
first day where women does not improve.
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Do not exist.
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Well Ordering Principle.
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   first day where women does not improve.
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Do not exist.
...and then induction...

\[ P(0) \land ((\forall n) (P(n) \implies P(n+1)) \equiv (\forall n \in N) P(n). \]
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in N) P(n). \]

**Thm:** For all \( n \geq 1 \), \( 8 \mid 3^{2n} - 1 \).
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n)). \]

**Thm:** For all \( n \geq 1 \), \( 8 \mid 3^{2n} - 1 \).

Induction on \( n \).
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n)). \]

**Thm:** For all \( n \geq 1 \), \( 8 \mid 3^{2n} - 1 \).

Induction on \( n \).

Base: \( 8 \mid 3^2 - 1 \).
...and then induction...

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**Thm:** For all \( n \geq 1, 8 \mid 3^{2^n} - 1. \)

Induction on \( n. \)

Base: \( 8 \mid 3^2 - 1. \)
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n). \]

**Thm:** For all \( n \geq 1 \), \( 8|3^{2n} - 1 \).

Induction on \( n \).

Base: \( 8|3^2 - 1 \).

Induction Hypothesis: True for some \( n \).
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n + 1)) \equiv (\forall n \in \mathbb{N}) P(n)). \]

**Thm:** For all \( n \geq 1 \), \( 8 | 3^{2n} - 1 \).

Induction on \( n \).

Base: \( 8 | 3^2 - 1 \).

Induction Hypothesis: True for some \( n \).

Induction Step:
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\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n)). \]

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Induction on \( n \).

Base: \( 8 \mid 3^2 - 1 \).

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Induction Step:
\[ 3^{2n+2} - 1 = \]
...and then induction...

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Induction on \( n \).

Base: \( 8 \mid 3^2 - 1 \).

Induction Hypothesis: True for some \( n \).

Induction Step:

\[
3^{2n+2} - 1 = 9(3^{2n}) - 1
\]
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in N) P(n). \]

**Thm:** For all \( n \geq 1 \), \( 8 \mid 3^{2n} - 1 \).

Induction on \( n \).

Base: \( 8 \mid 3^2 - 1 \).

Induction Hypothesis: True for some \( n \).

\[ (3^{2n} - 1 = 8d) \]

Induction Step:

\[ 3^{2n+2} - 1 = 9(3^{2n}) - 1 \] (by induction hypothesis)
...and then induction...

\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n). \]

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\[ = 9(8d + 1) - 1 \]
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\((3^{2n} - 1 = 8d)\)

Induction Step:

\[ 3^{2n+2} - 1 = 9(3^{2n}) - 1 \quad \text{ (by induction hypothesis)} \]
\[ = 9(8d + 1) - 1 \]
\[ = 72d + 8 \]
...and then induction...

\[ P(0) \land ((\forall n) (P(n) \implies P(n+1))) \equiv (\forall n \in \mathbb{N}) P(n). \]

**Thm:** For all \( n \geq 1 \), \( 8 \mid 3^{2n} - 1 \).

Induction on \( n \).

Base: \( 8 \mid 3^2 - 1 \).

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3^{2n+2} - 1 = 9(3^{2n}) - 1 \quad \text{(by induction hypothesis)} \\
= 9(8d + 1) - 1 \\
= 72d + 8 \\
= 8(9d + 1)
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**Thm:** For all \( n \geq 1, 8|3^{2n} - 1. \)

Induction on \( n. \)

Base: \( 8|3^2 - 1. \)

Induction Hypothesis: True for some \( n. \)

\( (3^{2n} - 1 = 8d) \)

Induction Step:

\[
3^{2n+2} - 1 = 9(3^{2n}) - 1 \quad \text{(by induction hypothesis)}
\]

\[= 9(8d + 1) - 1\]

\[= 72d + 8\]

\[= 8(9d + 1)\]

Divisible by 8.
\[ P(0) \land ((\forall n)(P(n) \implies P(n+1)) \equiv (\forall n \in \mathbb{N}) P(n)). \]

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Base: \( 8 \mid 3^2 - 1. \)

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\[ (3^{2n} - 1 = 8d) \]

Induction Step:
\[
3^{2n+2} - 1 = 9(3^{2n}) - 1 \quad \text{(by induction hypothesis)} \\
= 9(8d + 1) - 1 \\
= 72d + 8 \\
= 8(9d + 1)
\]

Divisible by 8.
Stable Marriage: a study in definitions and WOP.

\( n \)-men, \( n \)-women.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

Pairing.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list that contains every person of opposite gender.

**Pairing.**
Set of pairs $(m_i, w_j)$ containing all people *exactly* once.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
Set of pairs $(m_i, w_j)$ containing all people *exactly* once.
How many pairs?
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
- Set of pairs $(m_i, w_j)$ containing all people *exactly* once.
- How many pairs? $n$. 
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
Set of pairs $(m_i, w_j)$ containing all people *exactly* once.
How many pairs? $n$.
People in pair are **partners** in pairing.
n-men, n-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
Set of pairs \((m_i, w_j)\) containing all people exactly once.
How many pairs? \(n\).
People in pair are **partners** in pairing.

**Rogue Couple in a pairing.**
A \(m_j\) and \(w_k\) who like each other more than their partners
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

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**Pairing.**
Set of pairs \((m_i, w_j)\) containing all people *exactly* once.
How many pairs? \( n \).
People in pair are **partners** in pairing.

**Rogue Couple in a pairing.**
A \( m_j \) and \( w_k \) who like each other more than their partners

**Stable Pairing.**
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list that contains every person of opposite gender.

**Pairing.**

Set of pairs $(m_i, w_j)$ containing all people *exactly* once.

- How many pairs? $n$.
- People in pair are **partners** in pairing.

**Rogue Couple in a pairing.**

A $m_j$ and $w_k$ who like each other more than their partners

**Stable Pairing.**

Pairing with no rogue couples.
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
Set of pairs $(m_i, w_j)$ containing all people exactly once.
How many pairs? $n$.
People in pair are partners in pairing.

**Rogue Couple in a pairing.**
A $m_j$ and $w_k$ who like each other more than their partners

**Stable Pairing.**
Pairing with no rogue couples.

Does stable pairing exist?
Stable Marriage: a study in definitions and WOP.

$n$-men, $n$-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
Set of pairs $(m_i, w_j)$ containing all people *exactly* once.
How many pairs? $n$.
People in pair are partners in pairing.

**Rogue Couple in a pairing.**
A $m_j$ and $w_k$ who like each other more than their partners

**Stable Pairing.**
Pairing with no rogue couples.

Does stable pairing exist?
Stable Marriage: a study in definitions and WOP.

\( n \)-men, \( n \)-women.

Each person has completely ordered preference list contains every person of opposite gender.

**Pairing.**
- Set of pairs \((m_i, w_j)\) containing all people *exactly* once.
  - How many pairs? \( n \).
  - People in pair are **partners** in pairing.

**Rogue Couple in a pairing.**
- A \( m_j \) and \( w_k \) who like each other more than their partners

**Stable Pairing.**
- Pairing with no rogue couples.

Does stable pairing exist?
- No, for roommates problem.
Traditional Marriage Algorithm:

Each Day:

- Every man proposes to favorite woman who has not yet rejected him.
- Every woman rejects all but best men who proposes.

Useful Definitions:

- Man crosses off woman who rejected him.
- Woman's current proposer is "on string."

"Propose and Reject."

: Either men propose or women. But not both.

Traditional propose and reject where men propose.

Key Property: Improvement Lemma:

Every day, if man on string for woman, any future man on string is better.

Stability:

No rogue couple.

rogue couple (M,W) = ⇒ M proposed to W = ⇒ W ended up with someone she liked better than M.

Not rogue couple!
Traditional Marriage Algorithm:

Each Day:

TMA.
Traditional Marriage Algorithm:

Each Day:
Every man proposes to favorite woman who has not yet rejected him.
Traditional Marriage Algorithm:

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Traditional Marriage Algorithm:

Each Day:
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Useful Definitions:
  Man crosses off woman who rejected him.
  Woman’s current proposer is “on string.”
Traditional Marriage Algorithm:

Each Day:
Every man proposes to favorite woman who has not yet rejected him.
Every woman rejects all but best men who proposes.

Useful Definitions:
Man **crosses off** woman who rejected him.
Woman’s current proposer is **“on string.”**

“Propose and Reject.”
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Stability:
  No rogue couple.
Traditional Marriage Algorithm:

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rogue couple (M,W)
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Not rogue couple!
Optimality/Pessimal

Optimal partner if best partner in any stable pairing.
Optimality/Pessimal

Optimal partner if best partner in any stable pairing.
Not necessarily first in list.
Optimality/Pessimal

Optimal partner if best partner in any stable pairing. Not necessarily first in list. Possibly no stable pairing with that partner.

Man-optimal pairing is pairing where every man gets optimal partner.

**Thm:**

TMA produces male optimal pairing, $S$.

First man $M$ to lose optimal partner.

Better partner $W$ for $M$.

Different stable pairing $T$.

**TMA:** $M$ asked $W$ first!

There is $M'$ who bumps $M$ in TMA.

$W$ prefers $M'$.

$M'$ likes $W$ at least as much as optimal partner.

Not first bump.

$M'$ and $W$ is rogue couple in $T$.

**Thm:**

Woman pessimal.

Man optimal $\Rightarrow$ Woman pessimal.

Woman optimal $\Rightarrow$ Man pessimal.
Optimality/Pessimal

Optimal partner if best partner in any stable pairing.
Not necessarily first in list.
Possibly no stable pairing with that partner.
Optimality/Pessimal

Optimal partner if best partner in any *stable* pairing.
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Woman optimal $\Rightarrow$ Man pessimal.
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Possibly no stable pairing with that partner.

Man-optimal pairing is pairing where every man gets optimal partner.

Thm: TMA produces male optimal pairing, S.
First man $M$ to lose optimal partner.
Better partner $W$ for $M$.
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TMA: $M$ asked $W$ first!
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Man optimal $\implies$ Woman pessimal.
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Graphs

\[ G = (V, E) \]
...Graphs...

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\[ V - \text{ set of vertices.} \]
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...Graphs...

\[ G = (V, E) \]
- \( V \) - set of vertices.
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Directed: ordered pair of vertices.

Disconnected, Adjacent, Incident, Degree.
In-degree, Out-degree.

Thm: Sum of degrees is \( 2|E| \).

Edge is incident to 2 vertices.
Degree of vertices is total incidences.

Pair of Vertices are Connected: If there is a path between them.
Connected Component: maximal set of connected vertices.
Connected Graph: one connected component.
Graphs

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Algorithm:
- Take a walk.
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**Graph Algorithm: Eulerian Tour**

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*Recurse on connected components.*
*Put together.*
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Graph Types: Complete Graph.

Every edge present.

Very connected. Lots of edges: $n(n-1)/2$. 

$K_n$, $|V| = n$
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Trees.

Definitions:

- A connected graph without a cycle.
- A connected graph with \(|V| - 1\) edges.
- A connected graph where any edge removal disconnects it.
- An acyclic graph where any edge addition creates a cycle.

To tree or not to tree!

Minimally connected, minimum number of edges to connect.

Property: Can remove a single node and break into components of size at most \(|V|/2\).
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Hypercube

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$$G = (V, E)$$

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$G = (V, E)$

$|V| = \{0, 1\}^n,$

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Hypercubes. Really connected. $|V| \log |V|$ edges!
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Recursive Definition.

A 0-dimensional hypercube is a node labelled with the empty string of bits.
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An $n$-dimensional hypercube consists of a 0-subcube (1-subcube) which is a $n-1$-dimensional hypercube with nodes labelled $0x$ ($1x$) with the additional edges $(0x, 1x)$. 
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Hypercube: properties

Rudrata Cycle: cycle that visits every node.
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Eulerian?

FYI: Also cuts represent boolean functions.
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Large Cuts: Cutting off $k$ nodes needs $\geq k$ edges.
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Best cut? Cut apart subcubes:
Hypercube: properties

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Get from 000100 to 101000.
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Get from 000100 to 101000.

000100 $\rightarrow$ 100100 $\rightarrow$ 101100 $\rightarrow$ 101000
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- 000100 $\rightarrow$ 100100 $\rightarrow$ 101100 $\rightarrow$ 101000

Correct bits in string, moves along path in hypercube!
Rudrata Cycle: cycle that visits every node.
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Best cut? Cut apart subcubes: cuts off \( 2^n \) nodes with \( 2^{n-1} \) edges.

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Nice Paths between nodes.
Get from 000100 to 101000.
\[
000100 \rightarrow 100100 \rightarrow 101100 \rightarrow 101000
\]
Correct bits in string, moves along path in hypercube!
Rudrata Cycle: cycle that visits every node. Eulerian? If $n$ is even.

Large Cuts: Cutting off $k$ nodes needs $\geq k$ edges. Best cut? Cut apart subcubes: cuts off $2^n$ nodes with $2^{n-1}$ edges.

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Nice Paths between nodes.
Get from 000100 to 101000.
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Good communication network!
Modular Arithmetic...

Arithmetic modulo $m$.
Elements of equivalence classes of integers.
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$\{0, \ldots, m-1\}$
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and integer $i \equiv a \pmod{m}$
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if $i = a + km$ for integer $k$. 

...Modular Arithmetic...

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if $i = a + km$ for integer $k$.
or if the remainder of $i$ divided by $m$ is $a$. 

Can do calculations by taking remainders at the beginning, in the middle or at the end.

$58 + 32 = 90 = 6 \pmod{7}$

$58 + 32 = 2 + 4 = 6 \pmod{7}$

$58 + 32 = 2 + (-3) = -1 = 6 \pmod{7}$

Negative numbers work the way you are used to.
$-3 = 0 \pmod{7}$
$-3 = 7 \pmod{7}$
$-3 = 4 \pmod{7}$
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$x$ has inverse modulo $m$ if and only if $gcd(x, m) = 1$. 
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Group structures more generally.

Proof Idea:

\{0 \times x, \ldots, (m-1) \times x\} are distinct modulo $m$ if and only if $\gcd(x, m) = 1$.

Finding gcd:

$$\gcd(x, y) = \gcd(x, x - y) = \gcd(x, x \mod y).$$

Give recursive Algorithm!

Base Case? $\gcd(x, 0) = x$.

Extended-gcd($x, y$) returns $(d, a, b)$ such that $d = \gcd(x, y)$ and $d = ax + by$.

Multiplicative inverse of $(x, m)$.

$\text{egcd}(x, m) = (1, a, b)$

$a$ is inverse!

$$1 = ax + bm = ax \mod m.$$
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Many short answers.
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  Get at ideas that we study.
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Get at ideas that we study.
Know material well:

Know material fast, correct.
Know material medium, slower, less correct.
Know material not so well: Uh oh.

Some longer questions.
Proofs, algorithms, properties.
Not so much calculation.

Will post midterm from 4 years ago to get an idea.
Back when I was younger.
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If you sent me email about Midterm conflicts
Wrapup.

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If you sent me email about Midterm conflicts
Other arrangements.

satishr@cs.berkeley.edu

Private message on piazza.

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If you sent me email about Midterm conflicts
   Other arrangements.
   Should have received an email today from me.

Other issues....
satishr@cs.berkeley.edu
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