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Brief Comment.

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Math and Computer Science

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Efficient methods to construct objectes important in CS.

Lecture 17

What’s to come?

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What's to come? Probability.

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A bag contains:

Lecture 17

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What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

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What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

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Count blue. Count total. Divide.

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Today:

Lecture 17

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Today: Counting!

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Today: Counting!

Later: Probability.

Lecture 17

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Today: Counting!

Later: Probability. Professor Walrand.

Outline

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn’t matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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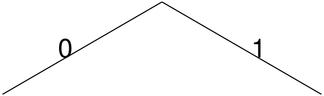

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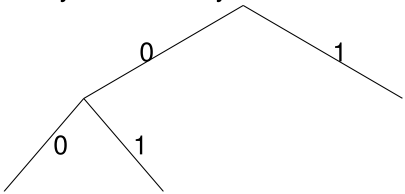

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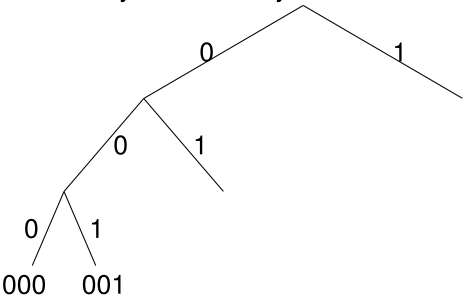

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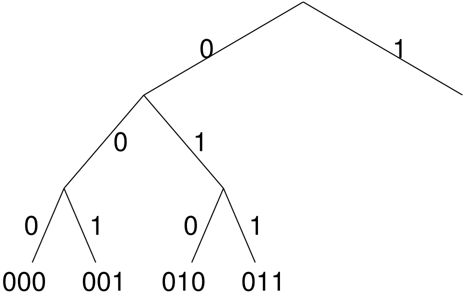

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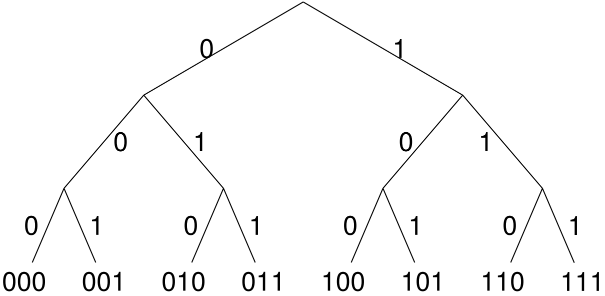

8 leaves which is $2 \times 2 \times 2$.

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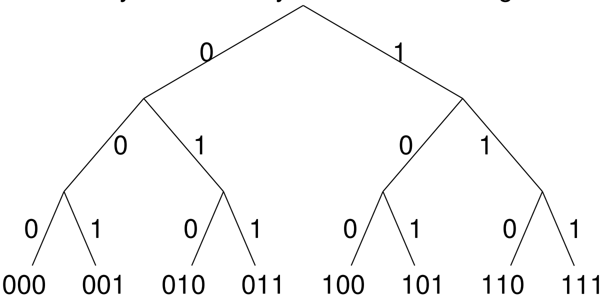

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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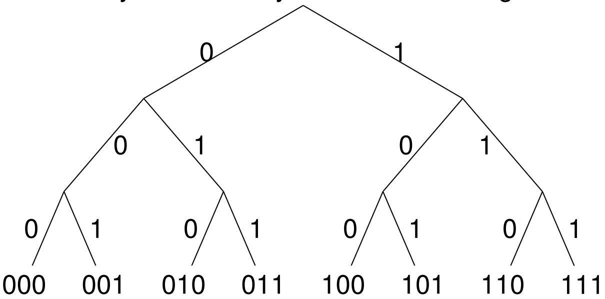

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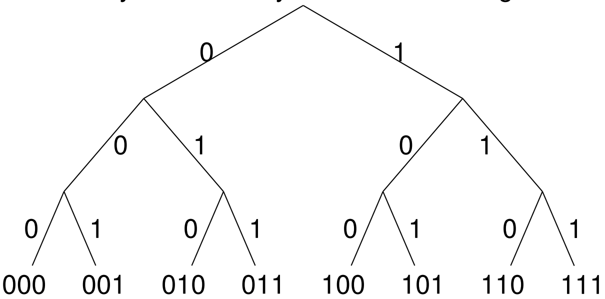

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First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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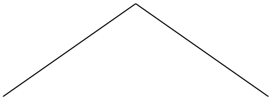

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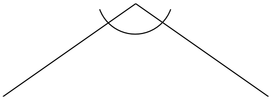

n_1

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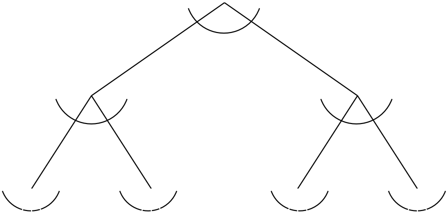

n_1

$\times n_2$

$\times n_3$

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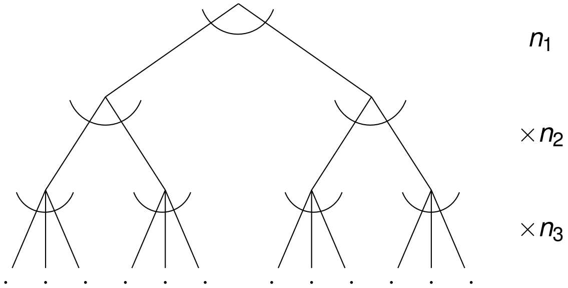

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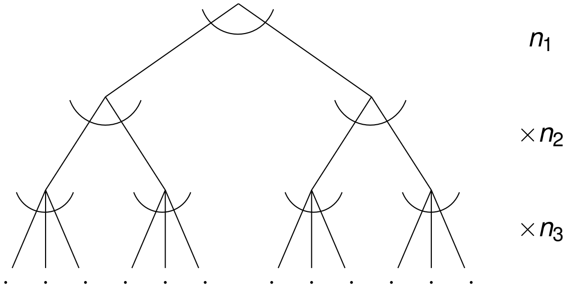

In picture, $2 \times 2 \times 3 = 12$!

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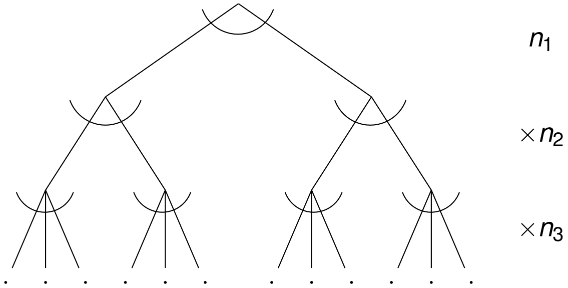

In picture, $2 \times 2 \times 3 = 12$!

Using the first rule..

How many outcomes possible for k coin tosses?

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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m^n

Functions, polynomials.

How many functions f mapping S to T?

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How many polynomials of degree d modulo p?

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Permutations.

[1] By definition: $0! = 1$.

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How many 10 digit numbers **without repeating a digit**?

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Permutations.

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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... $n*(n-1)*(n-2) \cdot *(n-k) = \frac{n!}{(n-k)!}$.

[1] By definition: $0! = 1$.

Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third, ...

... $10 * 9 * 8 \cdots * 1 = 10!$.[1]

How many different samples of size k from n numbers **without replacement.**

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Number of orderings for a poker hand: 5!.

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

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Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

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Number of orderings for a poker hand: 5!.

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2]When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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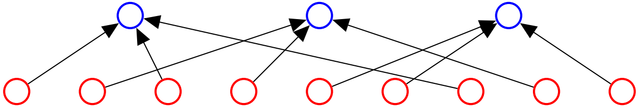

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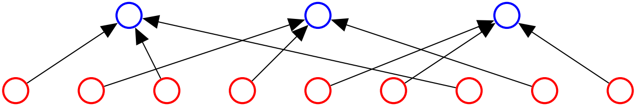

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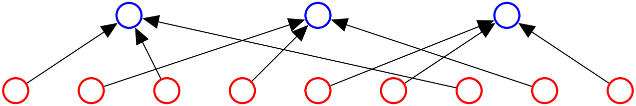

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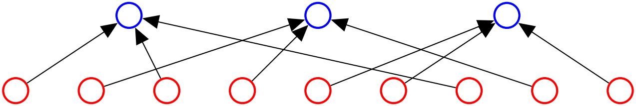

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

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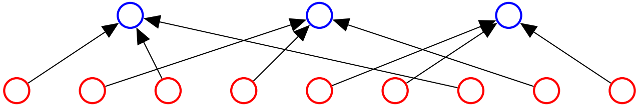

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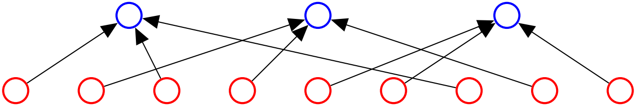

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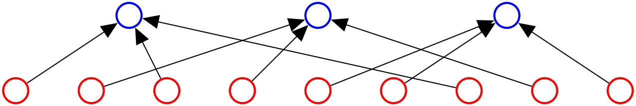

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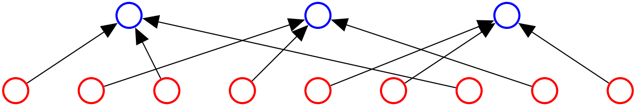

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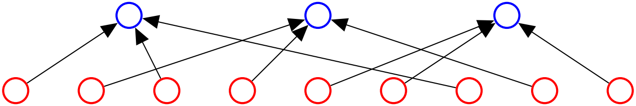

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How many poker deals?

Ordered to unordered.

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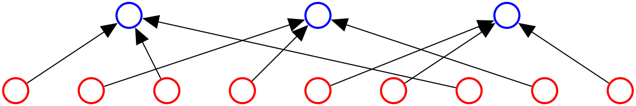

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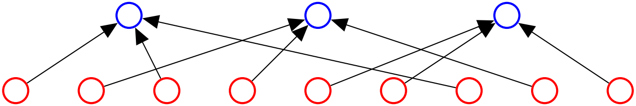

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How many poker hands per deal?

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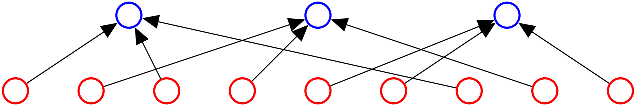

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How many poker hands per deal? Map each deal to ordered deal.

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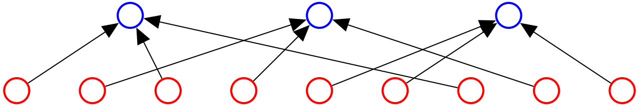

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How many poker hands per deal? Map each deal to ordered deal. $5!$

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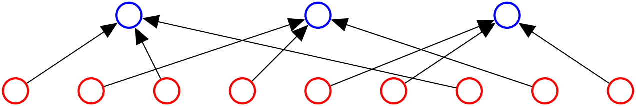

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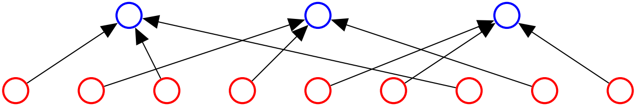

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Choose 2 out of n?

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Choose 2 out of n?

$$\frac{n \times (n-1)}{}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)! \qquad\quad}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)! \times k!}$$

Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

Simple Practice.

How many orderings of letters of CAT?

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How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways to choose second, 1 for last.

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$\implies 3 \times 2 \times 1$

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How many orderings of the letters in ANAGRAM?

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Ordered, except for A!

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Ordered, except for A!

total orderings of 7 letters.

Simple Practice.

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total orderings of 7 letters. 7!
total "extra counts" or orderings of two A's?

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How many orderings of MISSISSIPPI?

Simple Practice.

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How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.

Simple Practice.

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How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.
11 letters total!

Simple Practice.

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Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.
11 letters total!
11! ordered objects!

Simple Practice.

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total orderings of 7 letters. 7!
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Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.
11 letters total!
11! ordered objects!
$4! \times 4! \times 2!$ ordered objects per “unordered object”

Simple Practice.

How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways to choose second, 1 for last.

$\implies 3 \times 2 \times 1 = 3!$ orderings

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters. 7!
total "extra counts" or orderings of two A's? 3!

Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.
11 letters total!
11! ordered objects!
$4! \times 4! \times 2!$ ordered objects per "unordered object"

$\implies \frac{11!}{4!4!2!}$.

Sampling...

Sample k items out of n

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Without replacement:

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Order matters:

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Order matters: $n \times$

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Order matters: $n \times n-1 \times n-2 \ldots$

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Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

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  - Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
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    - Second Rule: divide by number of orders

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Order matters: $n \times n$

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How do we deal with this mess?!?!

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one ordered set: (B, B, B, B, B).

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Well, we can list the possibilities.
$0+5$, $1+4$,$2+3$, $3+2$, $4+1$, $5+0$.

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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

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How many different 5 star and 2 bar diagrams?

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$$\binom{n+k-1}{n-1}$$

Summary.

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Sample without replacement and order doesn't matter: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.
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Sample with replacement and order doesn't matter: $\binom{k+n-1}{n}$.

See you on Friday.