CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

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Add $99a + 11b$ to both sides.

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$\square$ Direct proof of $P \implies Q$: Assumed P: $11|a - b + c$.

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$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$.

$\square$ Direct proof of $P \implies Q$: Assumed P: $11|a - b + c$. Proved Q: $11|n$.

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

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- Proof assumed no primes *in between.*

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Proof: First a lemma...
Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Thus, in this case, we have irrational x and y with a rational x^y (i.e., 2).

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Thus, in this case, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

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Be careful.

Theorem: $3 = 4$

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$P \implies Q$ does not mean $Q \implies P$.

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And finally. Have a nice weekend!!