CS70: Jean Walrand: Lecture 39.

Final - Probability Review

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- True or False
- Some Key Results
- Sample Problems
- Common Mistakes

True or False

- Ω and A are independent.

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[2] $Pr[|X - E[X]| > a] \leq \frac{var[X]}{a^2}$

[3] $Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$

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[6] $\sum_y y Pr[Y = y | X = x]$

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- Chernoff

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- Jensen

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- Jensen (4)
- MMSE (6)
- Projection property

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- Chebyshev

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- Chebyshev (2)
- LLSE

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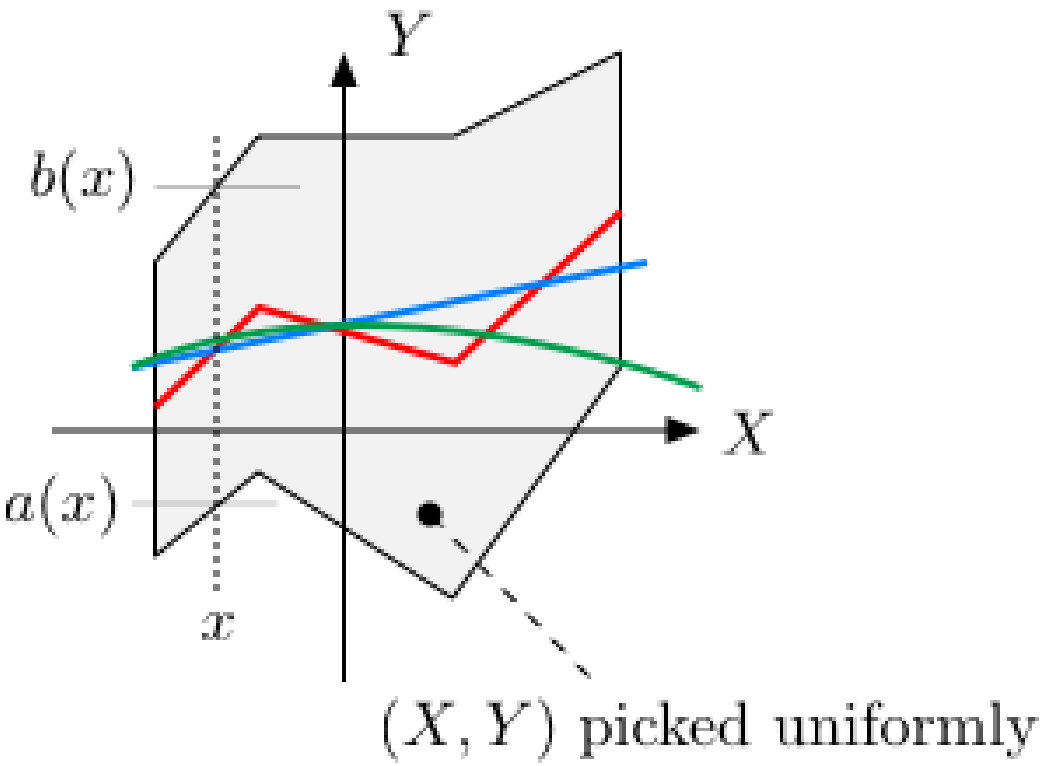
Conditional Expectation

Which is $E[Y|X]$? Blue, red or green?



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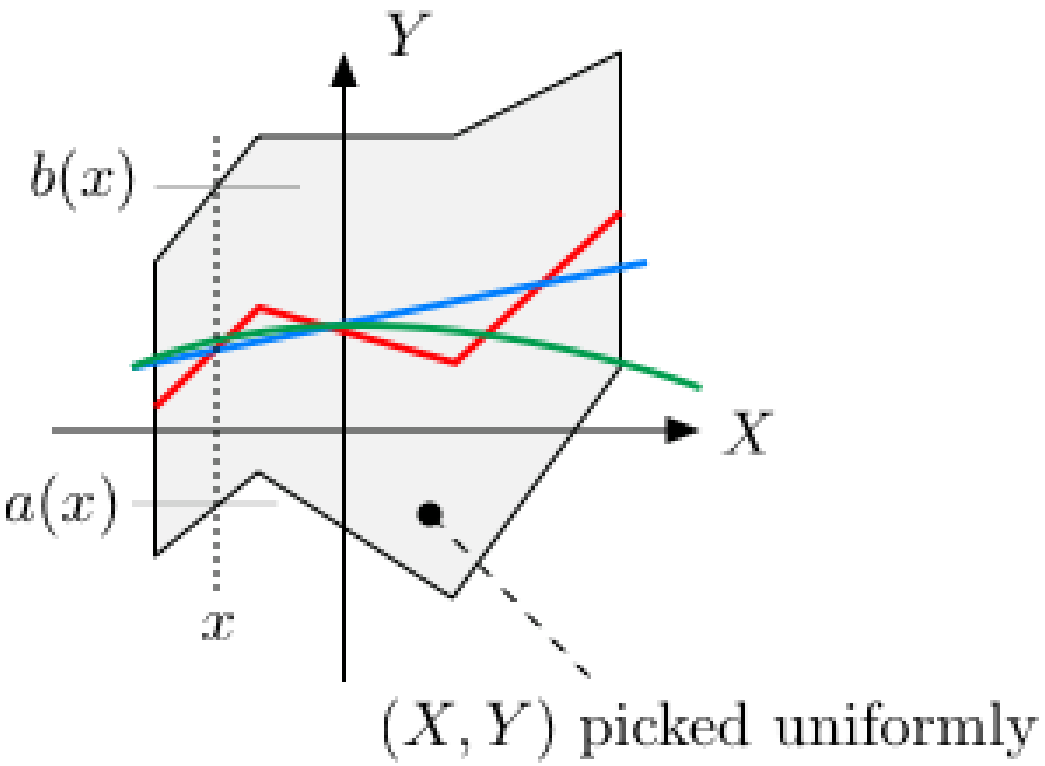

Answer: Red.

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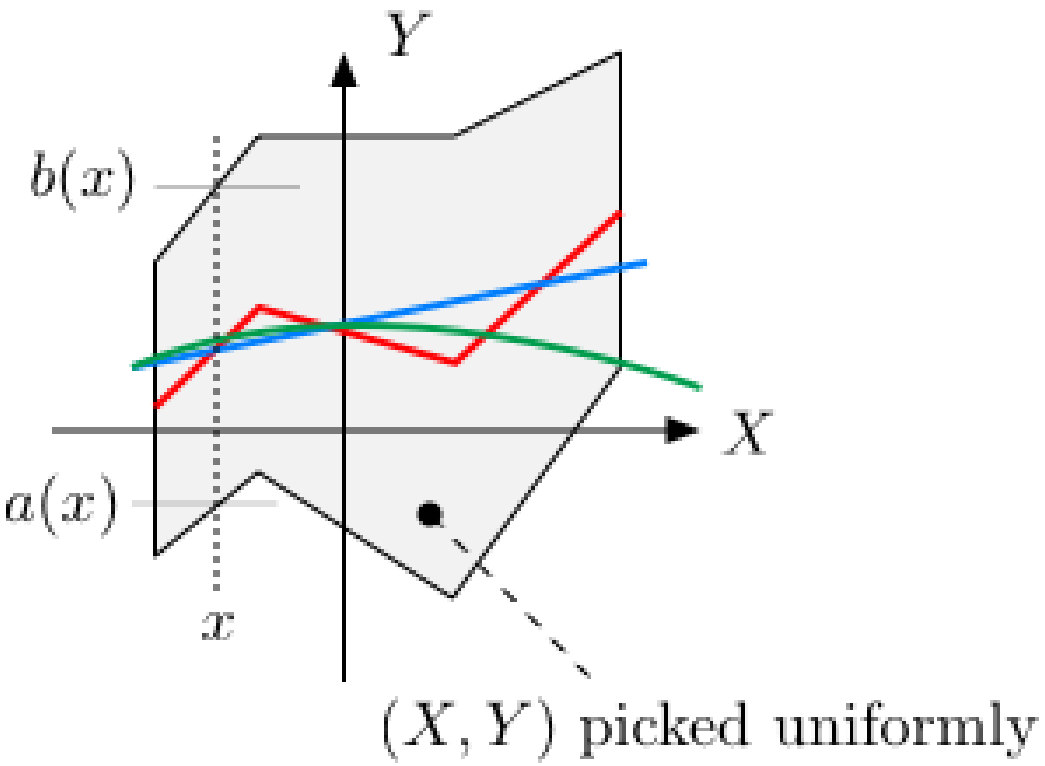

Answer: Red.
Given $X = x, Y = U[a(x), b(x)]$.

Conditional Expectation

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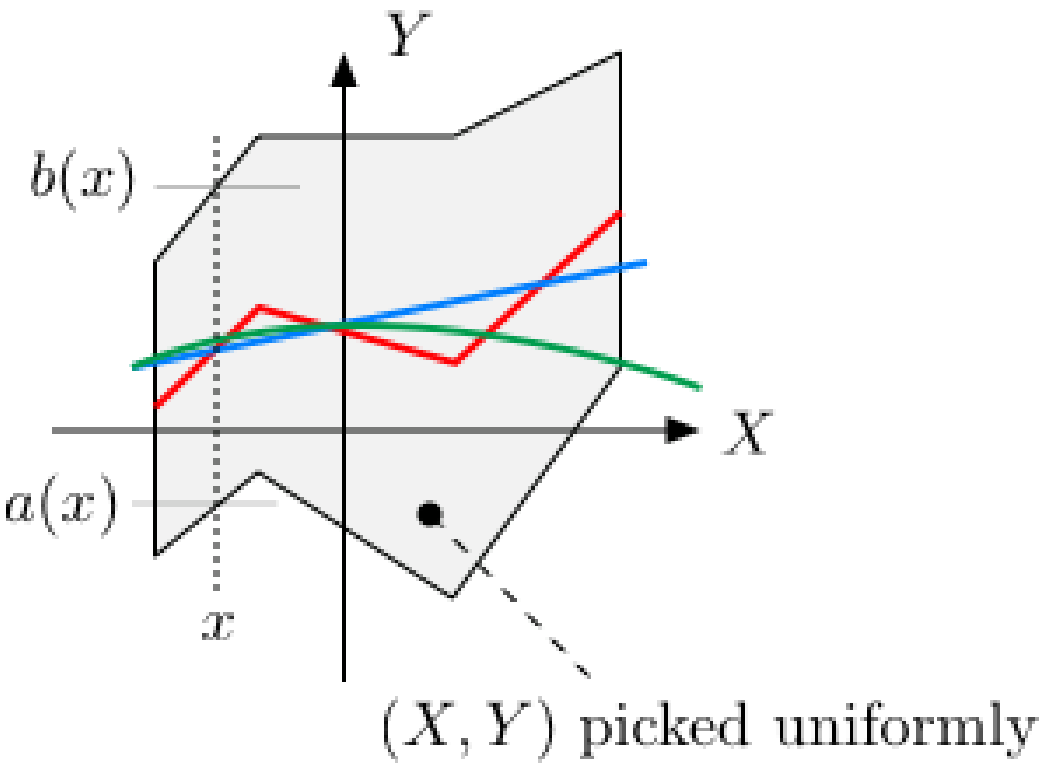
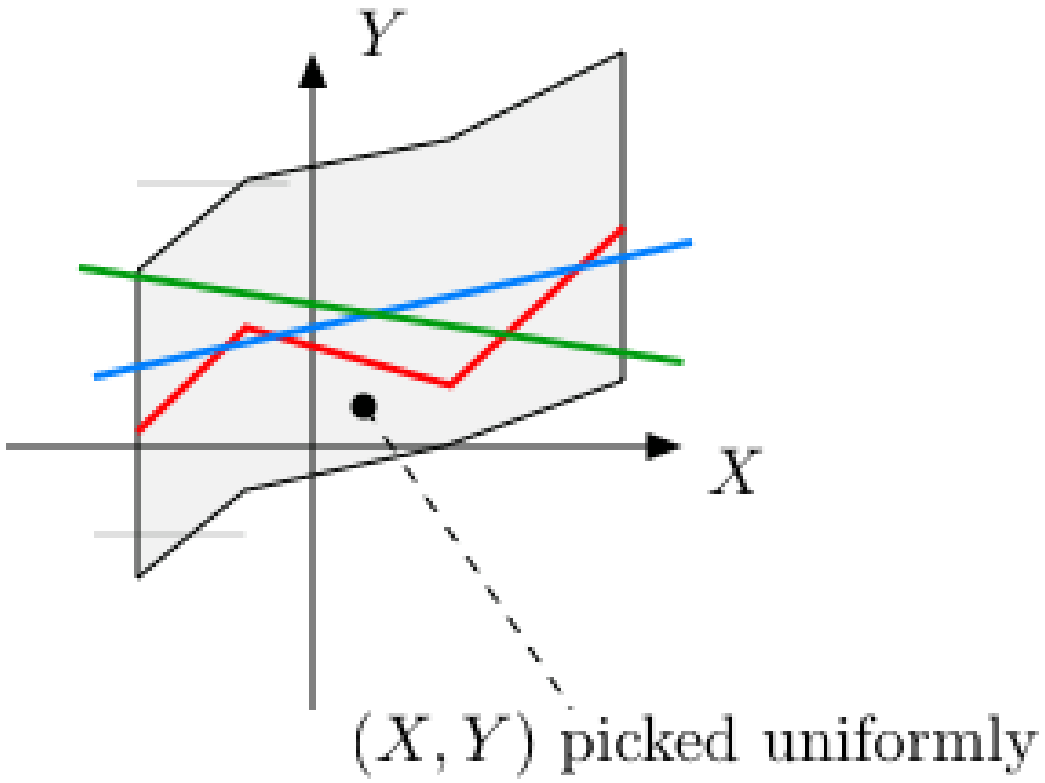


Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x)+b(x)}{2}$.

Linear Regression

Which is $L[Y|X]$? Blue, red or green?



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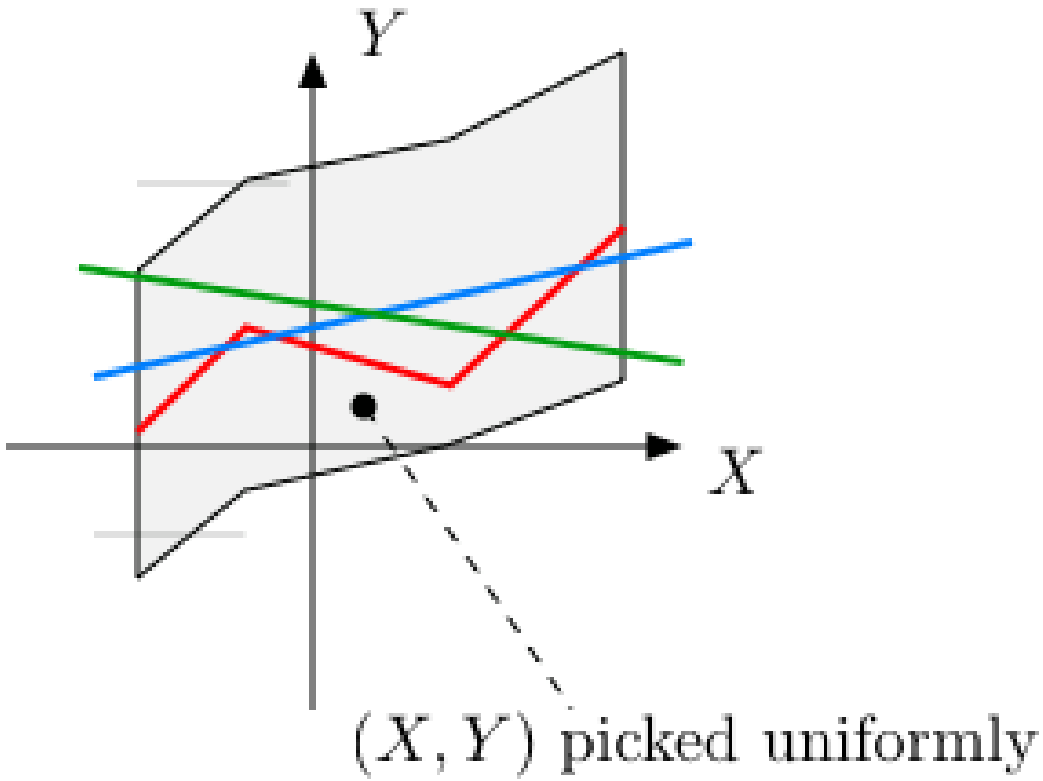

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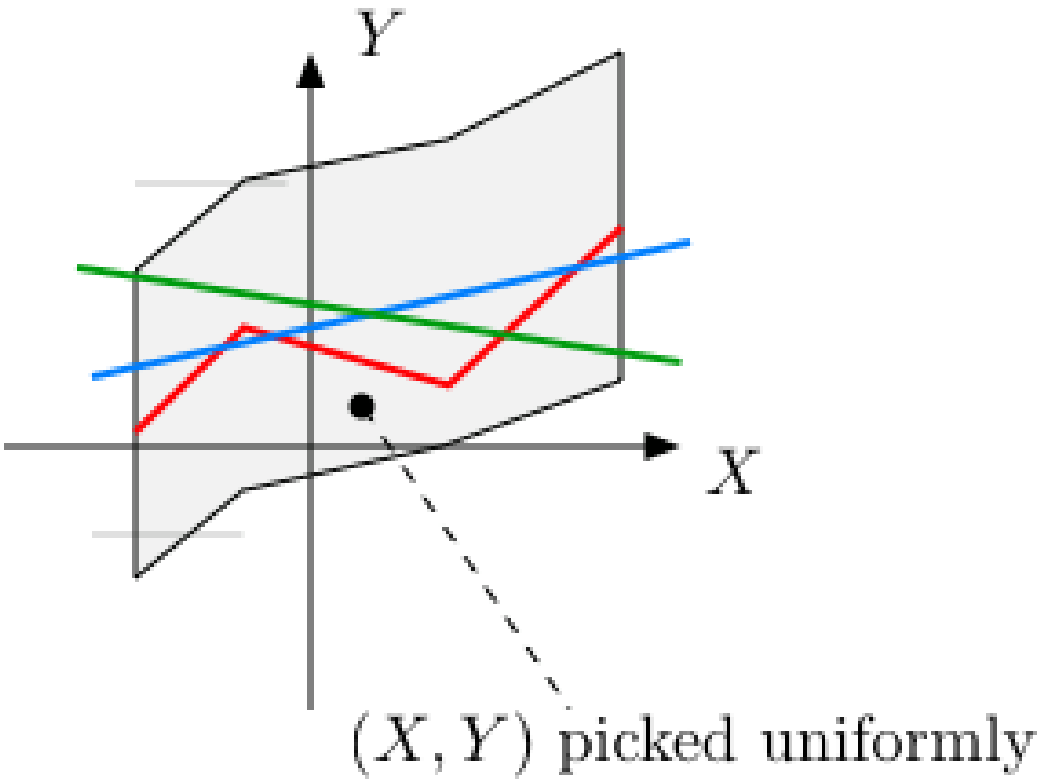

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Cannot be red (not a straight line).

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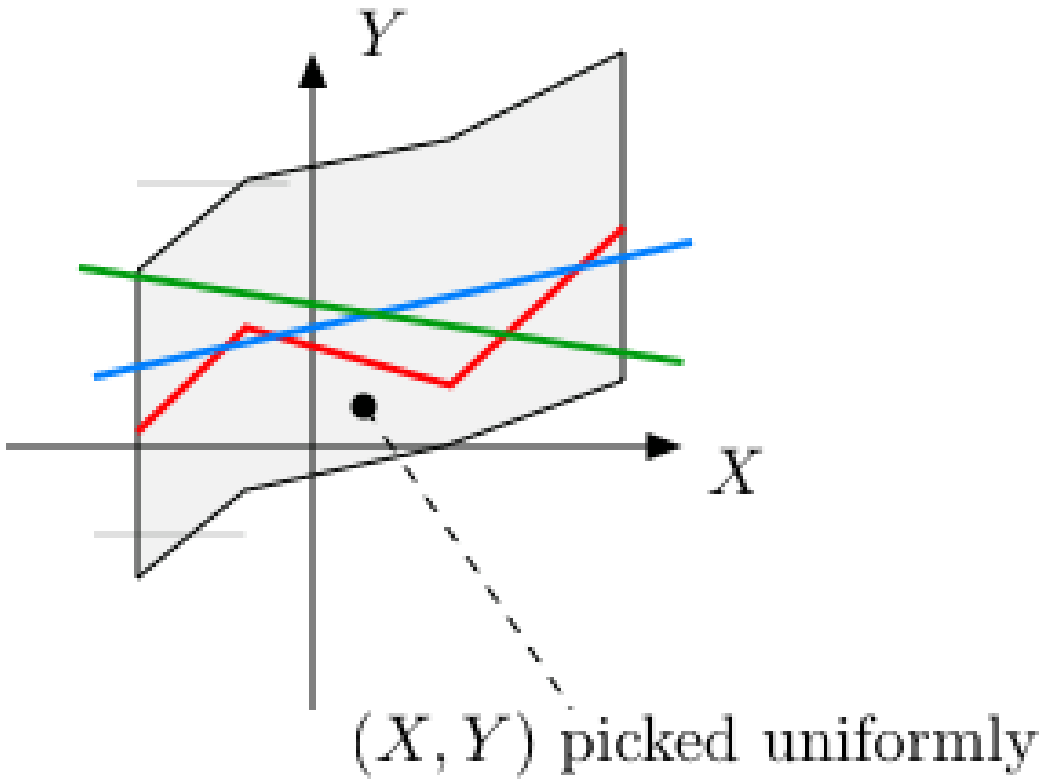

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Cannot be red (not a straight line).
Cannot be green: X and Y are clearly positively correlated.

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Clearly, negatively.

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$$f_Z(z) = \exp\{-z\} + 2\exp\{-2z\} - 3\exp\{-3z\}.$$

Hence,

$$E[Z] = \int_0^\infty z f_Z(z) dz = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}.$$

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Also, $Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.
Moreover, $E[V] = \int_0^\infty x\lambda \exp\{-\lambda x\} dx = \lambda^{-1}, var(V) = \lambda^{-2}$.

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We used $Pr[Z \in [x, x+\delta]] \approx f_Z(x)\delta$ and given A one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3\exp\{-3x\}$.

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- $E[X] = E[X|A] + E[X|\bar{A}]$. No.
- $\sum_{n=0}^{\infty} a^n = 1/a$. No.
- CS70 is difficult. No.
- I will do poorly on the final.

Common Mistakes

- $\Omega = \{1,2,3\}$. Define X, Y with $cov(X,Y) = 0$ and X, Y not independent.

  Let $X = 0, Y = 1$. No: They are independent.

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