Final - Probability Review
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- True or False
- Some Key Results
- Sample Problems
- Common Mistakes
True or False

- $\Omega$ and $A$ are independent.
True or False

- $\Omega$ and $A$ are independent.  True
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. False:
- $Pr[A \backslash B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\Rightarrow var(X_1 + \cdots + X_n) = var(X_1)$. False:
- $Pr(|X - a| \geq b) \leq E[(X - a)^2]^{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\Rightarrow X_1 + \cdots + X_n - nE[X_1] \rightarrow N(0, \sigma^2(X_1))$. False:
- $X = \text{Exponential}(\lambda) \Rightarrow Pr[X > 5 | X > 3] = Pr[X > 2]$. True:
- $\exp\{-\lambda \cdot 5\} \cdot \exp\{-\lambda \cdot 3\} = \exp\{-\lambda \cdot 2\}$. False:
True or False

- Ω and A are independent.  **True**
- \( Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B] \).  **True**
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
True or False

- \( \Omega \) and \( A \) are independent. \textbf{True}
- \( \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \). \textbf{True}
- \( \Pr[A \setminus B] \geq \Pr[A] - \Pr[B] \). \textbf{True}
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1)$. False

\[ \exp\{-\lambda x\} = \exp\{-\lambda y\} \]
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1 + \cdots + X_n}{n}\right) = var(X_1)$. False: $\times \frac{1}{n}$
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1+\cdots+X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}
- Pr[|X-a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True.
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1+\cdots+X_n}{n}\right) = var(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
True or False

- \( \Omega \) and \( A \) are independent.  **True**
- \( \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \).  **True**
- \( \Pr[A \setminus B] \geq \Pr[A] - \Pr[B] \).  **True**
- \( X_1, \ldots, X_n \) i.i.d.  \( \implies \) \( \text{var}(\frac{X_1+\cdots+X_n}{n}) = \text{var}(X_1) \).  **False**: \( \times \frac{1}{n} \)
- \( \Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \).  **True**
- \( X_1, \ldots, X_n \) i.i.d.  \( \implies \frac{X_1+\cdots+X_n-nE[X_1]}{n\sigma(X_1)} \to \mathcal{N}(0,1) \).
True or False

- \( \Omega \) and \( A \) are independent. True
- \( \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \). True
- \( \Pr[A \setminus B] \geq \Pr[A] - \Pr[B] \). True
- \( X_1, \ldots, X_n \) i.i.d. \( \implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1) \). False: \( \times \frac{1}{n} \)
- \( \Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \). True
- \( X_1, \ldots, X_n \) i.i.d. \( \implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \rightarrow \mathcal{N}(0,1) \). False: \( \sqrt{n} \)
True or False

- ▶ Ω and A are independent. True
- ▶ $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- ▶ $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- ▶ $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1+\ldots+X_n}{n}\right) = var(X_1)$. False: $\times \frac{1}{n}$
- ▶ $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- ▶ $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1+\ldots+X_n-nE[X_1]}{n\sigma(X_1)} \to \mathcal{N}(0,1)$. False: $\sqrt{n}$
- ▶ $X = Expo(\lambda) \implies Pr[X > 5|X > 3] = Pr[X > 2]$. True
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies var(\frac{X_1 + \cdots + X_n}{n}) = var(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \rightarrow \mathcal{N}(0,1)$. False: $\sqrt{n}$
- $X = Expo(\lambda) \implies Pr[X > 5|X > 3] = Pr[X > 2]$. True:
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
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- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \to N(0,1)$. False: $\sqrt{n}$
- $X = \text{Expo}(\lambda) \implies Pr[X > 5|X > 3] = Pr[X > 2]$. True:
  $$\frac{\exp{-\lambda 5}}{\exp{-\lambda 3}} = \exp{-\lambda 2}.$$
Correct or not?

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
Correct or not?

- \([A_n - 2\sigma \frac{1}{n}, A_n + 2\sigma \frac{1}{n}] = 95\%\text{-CI for } \mu\). No
Correct or not?

- \([A_n - 2\sigma_n, A_n + 2\sigma_n] = 95\%-\text{CI for } \mu\). No
- \([A_n - 2\sigma_n^{1/n}, A_n + 2\sigma_n^{1/n}] = 95\%-\text{CI for } \mu\). Yes
Correct or not?

- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%\text{-CI for } \mu. \) No
- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%\text{-CI for } \mu. \) Yes
Correct or not?

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
- If $0.3 < \sigma < 3$, then
  $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
Correct or not?

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. Yes
- If $0.3 < \sigma < 3$, then 
  $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. No
Correct or not?

- \([A_n - 2\sigma_n^1, A_n + 2\sigma_n^1] = 95\%\text{-CI for } \mu\). No
- \([A_n - 2\sigma_\sqrt{n}, A_n + 2\sigma_\sqrt{n}] = 95\%\text{-CI for } \mu\). Yes
- If \(0.3 < \sigma < 3\), then
  \([A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%\text{-CI for } \mu\). No
- If \(0.3 < \sigma < 3\), then
  \([A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%\text{-CI for } \mu\).
Correct or not?

- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu. \) No
- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu. \) Yes
- If 0.3 < \(\sigma\) < 3, then
  \([A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu. \) No
- If 0.3 < \(\sigma\) < 3, then
  \([A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu. \) Yes
Correct or not?

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. Yes
- If $0.3 < \sigma < 3$, then $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. No
- If $0.3 < \sigma < 3$, then $[A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu$. Yes
Match Items

[1] $Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)}$

[2] $Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2}$

[3] $Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$

[4] $g(\cdot)$ convex $\Rightarrow E[g(X)] \geq g(E[X])$

[5] $E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X])$

[6] $\sum_y yPr[Y = y | X = x]$

[7] $Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0,$

[8] $E[(Y - E[Y|X])h(X)] = 0.$
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0 \)

- Chernoff
Match Items

1. \[ \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \]

2. \[ \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \]

3. \[ \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \]

4. \[ g(\cdot) \text{ convex } \Rightarrow E[g(X)] \geq g(E[X]) \]

5. \[ E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \]

6. \[ \sum_y y \Pr[Y = y | X = x] \]

7. \[ \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \]

8. \[ E[(Y - E[Y|X])h(X)] = 0 \]

- Chernoff (3)
Match Items

[1] \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)

[2] \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)

[3] \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)

[4] \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)

[5] \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)

[6] \( \sum_y y Pr[Y = y | X = x] \)

[7] \( Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)

[8] \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[|\frac{X_1 + \cdots + X_n}{n} - E[X_1]| \geq \epsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN (7)
Match Items

1. $Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)}$.
2. $Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2}$.
3. $Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$.
4. $g(\cdot)$ convex $\Rightarrow E[g(X)] \geq g(E[X])$.
5. $E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X])$.
6. $\sum_{y} y Pr[Y = y | X = x]$.
7. $Pr[|\frac{X_1 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0$.
8. $E[(Y - E[Y|X])h(X)] = 0$.

- Chernoff (3)
- WLLN (7)
- Jensen
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_{y} y \Pr[Y = y | X = x] \)
7. \( \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0 \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)}(X - E[X]) \)
6. \( \sum_y yPr[Y = y|X = x] \)
7. \( Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE
Match Items

1. \[ Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \]
2. \[ Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \]
3. \[ Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \]
4. \[ g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \]
5. \[ E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \]
6. \[ \sum_y y Pr[Y = y | X = x] \]
7. \[ Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \rightarrow 0, \]
8. \[ E[(Y - E[Y|X])h(X)] = 0. \]

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\left\| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right\| \geq \varepsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. If \( g(\cdot) \) convex, then \( E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]). \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\frac{X_1 + \ldots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property (8)
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0 \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property (8)
- Chebyshev
Match Items

1. \[ \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \]
2. \[ \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \]
3. \[ \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \]
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \[ E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]). \]
6. \[ \sum_y y \Pr[Y = y|X = x] \]
7. \[ \Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0, \]
8. \[ E[(Y - E[Y|X])h(X)] = 0. \]

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE
Match Items

\[ Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \]

\[ Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \]

\[ Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \]

\[ g(\cdot) \text{ convex } \Rightarrow E[g(X)] \geq g(E[X]) \]

\[ E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \]

\[ \sum_y yPr[Y = y|X = x] \]

\[ Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \to 0, \]

\[ E[(Y - E[Y|X])h(X)] = 0. \]

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y Pr[Y = y|X = x] \)
7. \( Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \epsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- Chernoff (3)
- WLLN (7)
- Jensen (4)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov (1)
Conditional Expectation

Which is $E[Y|X]$? Blue, red or green?

![Diagram showing conditional expectation](image)
Conditional Expectation

Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
Conditional Expectation

Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. 
Which is $E[Y|X]$? Blue, red or green?

Answer: Red.

Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x) + b(x)}{2}$. 
Linear Regression

Which is $L[Y|X]$? Blue, red or green?

Answer: Blue. Cannot be red (not a straight line). Cannot be green: $X$ and $Y$ are clearly positively correlated.
Linear Regression

Which is $L[Y|X]$? Blue, red or green?

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→ Are $X$ and $Y$ positively, negatively, or uncorrelated?
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Are $X$ and $Y$ positively, negatively, or un-correlated?

Clearly, negatively.
A bag has $n$ red and $n$ blue balls. You pick two balls (no replacement). Let $X = 1$ if ball 1 is red and $X = -1$ otherwise. Define $Y$ likewise for ball 2.

→ Are $X$ and $Y$ positively, negatively, or uncorrelated? Clearly, negatively.

→ Calculate $\text{cov}(X, Y)$. 

\[
\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X] \mathbb{E}[Y],
\]

by symmetry

\[
\mathbb{E}[X] = 0
\]

\[
\mathbb{E}[XY] = \Pr[X = Y] - \Pr[X \neq Y] = \frac{n-1}{2n-1}
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Indeed, $\text{var}(X) = 1$, obviously!
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E[X] = E[Y], \text{ by symmetry}
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Since $X$ takes only two values, any $g(X)$ is linear in $X$. 
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Thus,

$$E[Y|X] = (2\alpha - 1)X = -\frac{1}{2n-1}X.$$
Let $X, Y, Z$ be i.i.d.
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$\rightarrow$ Compute $E[X|X + Y + Z]$. 
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Answer: $(X + Y + Z)/3$. 
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Continuous RV

Let $X = Expo(1)$ and $Y = Expo(2)$ be independent.
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Let $Z = \max\{X, Y\}$. 

Continuous RV

Let $X = Expo(1)$ and $Y = Expo(2)$ be independent.
Let $Z = \max\{X, Y\}$. Calculate $E[Z]$. 

Recall: $V = Expo(\lambda) \implies f_V(x) = \lambda e^{-\lambda x} 1_{\{x \geq 0\}}$.
Also, $\Pr[V \leq x] = 1 - e^{-\lambda x}$ for $x \geq 0$.
Moreover, $E[V] = \int_0^\infty x \lambda e^{-\lambda x} dx = \frac{1}{\lambda} - 1$, $\text{var}(V) = \lambda - 2$.

For $z > 0$, one has $\Pr[\max\{X, Y\} \leq z] = \Pr[X \leq z, Y \leq z] = \Pr[X \leq z] \Pr[Y \leq z] = (1 - e^{-z}) (1 - e^{-2z}) = 1 - e^{-z} - e^{-2z} + e^{-3z}$.

Thus, for $z > 0$, taking the derivative, $f_Z(z) = e^{-z} + 2e^{-2z} - 3e^{-3z}$.
Hence, $E[Z] = \int_0^\infty z f_Z(z) dz = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$. 
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Continuous RV

Let $X = \text{Expo}(1)$ and $Y = \text{Expo}(2)$ be independent.
Let $Z = \max\{X, Y\}$. Calculate $E[Z]$.
Recall: $V = \text{Expo}(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}$.
Also, $Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.
Moreover, $E[V] = \int_0^\infty x \lambda \exp\{-\lambda x\} dx = \lambda^{-1}, \text{var}(V) = \lambda^{-2}$.

For $z > 0$, one has

\[
Pr[\max\{X, Y\} \leq z] = Pr[X \leq z, Y \leq z] = Pr[X \leq z] Pr[Y \leq z].
\]
\[
= (1 - \exp\{-z\})(1 - \exp\{-2z\})
\]
\[
= 1 - \exp\{-z\} - \exp\{-2z\} + \exp\{-3z\}.
\]

Thus, for $z > 0$, taking the derivative,

\[
f_Z(z) = \exp\{-z\} + 2 \exp\{-2z\} - 3 \exp\{-3z\}.
\]
Continuous RV

Let $X = \text{Expo}(1)$ and $Y = \text{Expo}(2)$ be independent.
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Moreover, $E[V] = \int_0^\infty x\lambda \exp\{-\lambda x\} \, dx = \lambda^{-1}, \text{var}(V) = \lambda^{-2}$.

For $z > 0$, one has

$Pr[\max\{X, Y\} \leq z] = Pr[X \leq z, Y \leq z] = Pr[X \leq z] Pr[Y \leq z]$.
$= (1 - \exp\{-z\})(1 - \exp\{-2z\})$
$= 1 - \exp\{-z\} - \exp\{-2z\} + \exp\{-3z\}$.

Thus, for $z > 0$, taking the derivative,

$f_Z(z) = \exp\{-z\} + 2\exp\{-2z\} - 3\exp\{-3z\}$.

Hence,

$E[Z] = \int_0^\infty zf_Z(z) \, dz = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$. 
Let \( X = Expo(1) \) and \( Y = Expo(2) \) be independent.
Let $X = Expo(1)$ and $Y = Expo(2)$ be independent.
Let $W = \min\{X, Y\}$.
Let $X = Expo(1)$ and $Y = Expo(2)$ be independent. Let $W = \min\{X, Y\}$. Calculate $E[W]$. 
Let \( X = \text{Expo}(1) \) and \( Y = \text{Expo}(2) \) be independent. Let \( W = \min\{X, Y\} \). Calculate \( E[W] \).

Recall: \( V = \text{Expo}(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\} \).
Also, \( \Pr[V \leq x] = 1 - \exp\{-\lambda x\} \) for \( x \geq 0 \).
Moreover, \( E[V] = \int_0^\infty x \lambda \exp\{-\lambda x\} dx = \lambda^{-1}, \text{var}(V) = \lambda^{-2} \).

For \( z > 0 \), one has

\[
\Pr[\min\{X, Y\} \geq z] = \exp\{-3z\}.
\]
Continuous RV

Let $X = Expo(1)$ and $Y = Expo(2)$ be independent. Let $W = \min\{X, Y\}$. Calculate $E[W]$.

Recall: $V = Expo(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}$.

Also, $Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.

Moreover, $E[V] = \int_0^\infty x\lambda \exp\{-\lambda x\}dx = \lambda^{-1}$, $\text{var}(V) = \lambda^{-2}$.

For $z > 0$, one has

$$Pr[\min\{X, Y\} \geq z] = Pr[X \geq z, Y \geq z] =$$
Continuous RV

Let $X = Expo(1)$ and $Y = Expo(2)$ be independent. Let $W = \min\{X, Y\}$. Calculate $E[W]$.

Recall: $V = Expo(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\}$. Also, $Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.

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For $z > 0$, one has

$$Pr[\min\{X, Y\} \geq z] = Pr[X \geq z, Y \geq z] = Pr[X \geq z]Pr[Y \geq z].$$
Continuous RV

Let $X = Expo(1)$ and $Y = Expo(2)$ be independent. Let $W = \min\{X, Y\}$. Calculate $E[W]$.

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For $z > 0$, one has

$$Pr[\min\{X, Y\} \geq z] = Pr[X \geq z, Y \geq z] = Pr[X \geq z]Pr[Y \geq z].$$

$$= \exp\{-z\} \exp\{-2z\}$$
Let $X = Expo(1)$ and $Y = Expo(2)$ be independent. Let $W = \min\{X, Y\}$. Calculate $E[W]$.

Recall: $V = Expo(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}$. Also, $Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.

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For $z > 0$, one has

$$Pr[\min\{X, Y\} \geq z] = Pr[X \geq z, Y \geq z] = Pr[X \geq z]Pr[Y \geq z].$$

$$= \exp\{-z\}\exp\{-2z\} = \exp\{-3z\}.$$
Continuous RV

Let $X = \text{Expo}(1)$ and $Y = \text{Expo}(2)$ be independent.
Let $W = \min\{X, Y\}$. Calculate $E[W]$.
Recall: $V = \text{Expo}(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}$.
Also, $\Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.
Moreover, $E[V] = \int_0^\infty x\lambda \exp\{-\lambda x\} dx = \lambda^{-1}, \text{var}(V) = \lambda^{-2}$.

For $z > 0$, one has

$$\Pr[\min\{X, Y\} \geq z] = \Pr[X \geq z, Y \geq z] = \Pr[X \geq z]\Pr[Y \geq z].$$
$$= \exp\{-z\} \exp\{-2z\} = \exp\{-3z\}.$$ 

Thus, $W = \text{Expo}(3)$. 
Let $X = \text{Expo}(1)$ and $Y = \text{Expo}(2)$ be independent.
Let $W = \min\{X, Y\}$. Calculate $E[W]$.
Recall: $V = \text{Expo}(\lambda) \implies f_V(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}$.  
Also, $\Pr[V \leq x] = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.
Moreover, $E[V] = \int_{0}^{\infty} x \lambda \exp\{-\lambda x\} dx = \lambda^{-1}$, $\text{var}(V) = \lambda^{-2}$.

For $z > 0$, one has

$$
\Pr[\min\{X, Y\} \geq z] = \Pr[X \geq z, Y \geq z] = \Pr[X \geq z] \Pr[Y \geq z].
$$

$$
= \exp\{-z\} \exp\{-2z\} = \exp\{-3z\}.
$$

Thus, $W = \text{Expo}(3)$. Hence, $E[W] = \frac{1}{3}$.  

Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $Exp(1)$ and w.p. 1/2, they are i.i.d. $Exp(3)$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

Let $A$ be the event that $X, Y$ are $Exp(1)$.

Then, $\Pr[A|B] = \frac{1}{2} \Pr[B|A] + \frac{1}{2} \Pr[B|\overline{A}]$

$$= \exp\left(-x\right) \delta \exp\left(-x\right) \delta + 3 \exp\left(-3x\right) \delta$$

$$= \exp\left(-2x\right) \delta + \exp\left(-2x\right) \delta + 3 \exp\left(-3x\right) \delta = e^2x^3 + e^2x.$$  

Now, $E[Y|X=x] = E[Y|A] \Pr[A|X=x] + E[Y|\overline{A}] \Pr[\overline{A}|X=x]$. 

We used $\Pr[Z \in [x, x + \delta]] \approx f_Z(x) \delta$ and given $A$ one has $f_X(x) = \exp\left(-x\right)$ whereas given $\overline{A}$ one has $f_X(x) = 3 \exp\left(-3x\right)$. 

Continuous RV and Bayes’ Rule
W.p. 1/2, X, Y are i.i.d. Expo(1) and w.p. 1/2, they are i.i.d. Expo(3).

Calculate $E[Y|X = x]$. 

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

Let $A$ be the event that $X, Y$ are Expo(1).

Then, $\Pr[A|B] = \left(\frac{1}{2}\right) \Pr[B|A] \left(\frac{1}{2}\right) \Pr[B|\bar{A}] + \left(\frac{1}{2}\right) \Pr[B|\bar{A}]$.

We used $\Pr[Z \in [x, x + \delta]] \approx f_Z(x) \delta$ and given $A$ one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3\exp\{-3x\}$. 


Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $Expo(1)$ and w.p. 1/2, they are i.i.d. $Expo(3)$.

Calculate $E[Y|X = x]$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$. 

We used $\Pr[Z \in [x, x + \delta]] \approx f_Z(x) \delta$ and given $A$ one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3 \exp\{-3x\}$. 

Continuous RV and Bayes’ Rule

W.p. 1/2, \(X, Y\) are i.i.d. \(\text{Expo}(1)\) and w.p. 1/2, they are i.i.d. \(\text{Expo}(3)\).

Calculate \(E[Y \mid X = x]\).

Let \(B\) be the event that \(X \in [x, x + \delta]\) where \(0 < \delta \ll 1\).

Let \(A\) be the event that \(X, Y\) are \(\text{Expo}(1)\).
Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $\text{Expo}(1)$ and w.p. 1/2, they are i.i.d. $\text{Expo}(3)$.

Calculate $E[Y|X = x]$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

Let $A$ be the event that $X, Y$ are $\text{Expo}(1)$.

Then,

$$Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]}$$
Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $\text{Expo}(1)$ and w.p. 1/2, they are i.i.d. $\text{Expo}(3)$.

Calculate $E[Y|X = x]$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

Let $A$ be the event that $X, Y$ are $\text{Expo}(1)$.

Then,

\[
Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\} \delta}{\exp\{-x\} \delta + 3 \exp\{-3x\} \delta}
\]
Continuous RV and Bayes’ Rule

W.p. 1/2, \( X, Y \) are i.i.d. Expo(1) and w.p. 1/2, they are i.i.d. Expo(3).

Calculate \( E[Y|X = x] \).

Let \( B \) be the event that \( X \in [x, x + \delta] \) where \( 0 < \delta \ll 1 \).

Let \( A \) be the event that \( X, Y \) are Expo(1).

Then,

\[
Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\}\delta}{\exp\{-x\}\delta + 3\exp\{-3x\}\delta}
\]

\[
= \frac{\exp\{-x\}}{\exp\{-x\} + 3\exp\{-3x\}}
\]
Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $Expo(1)$ and w.p. 1/2, they are i.i.d. $Expo(3)$.

Calculate $E[Y|X = x]$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

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Then,

$$Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\}\delta}{\exp\{-x\}\delta + 3\exp\{-3x\}\delta}$$

$$= \frac{\exp\{-x\}}{\exp\{-x\} + 3\exp\{-3x\}} = \frac{e^{2x}}{3 + e^{2x}}.$$
Continuous RV and Bayes’ Rule

W.p. 1/2, \( X, Y \) are i.i.d. \( \text{Expo}(1) \) and w.p. 1/2, they are i.i.d. \( \text{Expo}(3) \).

Calculate \( E[Y|X = x] \).

Let \( B \) be the event that \( X \in [x, x + \delta] \) where \( 0 < \delta \ll 1 \).

Let \( A \) be the event that \( X, Y \) are \( \text{Expo}(1) \).

Then,

\[
Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\} \delta}{\exp\{-x\} \delta + 3 \exp\{-3x\} \delta} = \frac{\exp\{-x\}}{\exp\{-x\} + 3 \exp\{-3x\}} = \frac{\exp2x}{3 + \exp2x}.
\]

Now,

\[
E[Y|X = x] = E[Y|A]Pr[A|X = x] + E[Y|\bar{A}]Pr[\bar{A}|X = x]
\]
Continuous RV and Bayes’ Rule

W.p. 1/2, \( X, Y \) are i.i.d. \( \text{Expo}(1) \) and w.p. 1/2, they are i.i.d. \( \text{Expo}(3) \).

Calculate \( E[Y|X = x] \).

Let \( B \) be the event that \( X \in [x, x + \delta] \) where \( 0 < \delta \ll 1 \).

Let \( A \) be the event that \( X, Y \) are \( \text{Expo}(1) \).

Then,
\[
Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\}\delta}{\exp\{-x\}\delta + 3\exp\{-3x\}\delta} = \frac{\exp\{-x\}}{\exp\{-x\} + 3\exp\{-3x\}} = \frac{e^{2x}}{3 + e^{2x}}.
\]

Now,
\[
E[Y|X = x] = E[Y|A]Pr[A|X = x] + E[Y|\bar{A}]Pr[\bar{A}|X = x]
= 1 \times Pr[A|X = x] + (1/3)Pr[\bar{A}|X = x]
\]
Continuous RV and Bayes’ Rule

W.p. 1/2, $X, Y$ are i.i.d. $\text{Expo}(1)$ and w.p. 1/2, they are i.i.d. $\text{Expo}(3)$.

Calculate $E[Y|X = x]$.

Let $B$ be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

Let $A$ be the event that $X, Y$ are $\text{Expo}(1)$.

Then,

$$Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]} = \frac{\exp\{-x\}\delta}{\exp\{-x\}\delta + 3\exp\{-3x\}\delta} = \frac{\exp\{-x\}}{\exp\{-x\} + 3\exp\{-3x\}} = \frac{e^{2x}}{3 + e^{2x}}.$$

Now,

$$E[Y|X = x] = E[Y|A]Pr[A|X = x] + E[Y|\bar{A}]Pr[\bar{A}|X = x]$$

$$= 1 \times Pr[A|X = x] + (1/3)Pr[\bar{A}|X = x]... = \frac{1 + e^{2x}}{3 + e^{2x}}.$$

We used $Pr[Z \in [x, x + \delta]] \approx f_Z(x)\delta$ and given $A$ one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3\exp\{-3x\}$. 
Rolling Dice

You roll a balanced die.
Rolling Dice

You roll a balanced die.

You start with $1.00.
Rolling Dice

You roll a balanced die.

You start with $1.00.

Every time you get a 6, your fortune is multiplied by 10.
Rolling Dice

You roll a balanced die.

You start with $1.00.

Every time you get a 6, your fortune is multiplied by 10.
Every time you do not get a 6, your fortune is divided by 2.
Rolling Dice

You roll a balanced die.

You start with $1.00.

Every time you get a 6, your fortune is multiplied by 10.
Every time you do not get a 6, your fortune is divided by 2.

Let $X_n$ be your fortune at the start of step $n$, 

$$E[X_{n+1}] = 10 \cdot P(6) + \frac{1}{2} \cdot P(\text{not 6})$$ 

$$E[X_{n+1}] = 10 \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{5}{6}$$ 

$$E[X_{n+1}] = \frac{11}{6} \approx 1.83$$ 

Hence, 

$$E[X_{n+1}] = \rho \cdot E[X_n]$$ 

for $n \geq 1$. 

Thus, 

$$E[X_n] = \rho^{n-1} E[X_1]$$ 

$$E[X_n] = \frac{11}{6}^{n-1}$$ 

$$E[X_n] \approx \left(\frac{11}{6}\right)^{n-1}$$
Rolling Dice

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You start with $1.00.
Every time you get a 6, your fortune is multiplied by 10.
Every time you do not get a 6, your fortune is divided by 2.
Let $X_n$ be your fortune at the start of step $n$,
Calculate $E[X_n]$. 
Rolling Dice

You roll a balanced die.

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Let $X_n$ be your fortune at the start of step $n$,

Calculate $E[X_n]$.

We have $X_1 = 1$. 
Rolling Dice

You roll a balanced die.
You start with $1.00.
Every time you get a 6, your fortune is multiplied by 10.
Every time you do not get a 6, your fortune is divided by 2.
Let $X_n$ be your fortune at the start of step $n$.
Calculate $E[X_n]$.

We have $X_1 = 1$. Also,

$$E[X_{n+1}|X_n] = X_n \times \left[10 \cdot \frac{1}{6} + 0.5 \times \frac{5}{6}\right]$$
Rolling Dice

You roll a balanced die.
You start with $1.00.
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$$E[X_{n+1}|X_n] = X_n \times \left[ 10 \frac{1}{6} + 0.5 \times \frac{5}{6} \right]$$

$$= \rho X_n, \rho = 10 \frac{1}{6} + 0.5 \times \frac{5}{6}$$
Rolling Dice

You roll a balanced die.

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Every time you get a 6, your fortune is multiplied by 10.
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Let $X_n$ be your fortune at the start of step $n$.

Calculate $E[X_n]$.

We have $X_1 = 1$. Also,

$$E[X_{n+1}|X_n] = X_n \times [10 \frac{1}{6} + 0.5 \times \frac{5}{6}]$$

$$= \rho X_n, \rho = 10 \frac{1}{6} + 0.5 \times \frac{5}{6} \approx 2.1.$$
Rolling Dice

You roll a balanced die.

You start with $1.00.

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Every time you do not get a 6, your fortune is divided by 2.

Let $X_n$ be your fortune at the start of step $n$.

Calculate $E[X_n]$.

We have $X_1 = 1$. Also,

$$E[X_{n+1} | X_n] = X_n \times \left[10 \frac{1}{6} + 0.5 \times \frac{5}{6}\right]$$

$$= \rho X_n, \quad \rho = 10 \frac{1}{6} + 0.5 \times \frac{5}{6} \approx 2.1.$$ 

Hence,

$$E[X_{n+1}] = \rho E[X_n], \quad n \geq 1.$$
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Calculate $E[X_n]$.

We have $X_1 = 1$. Also,

$$E[X_{n+1}|X_n] = X_n \times \left[10 \frac{1}{6} + 0.5 \times \frac{5}{6}\right]$$

$$= \rho X_n, \quad \rho = 10 \frac{1}{6} + 0.5 \times \frac{5}{6} \approx 2.1.$$ 

Hence,

$$E[X_{n+1}] = \rho E[X_n], \quad n \geq 1.$$ 

Thus,

$$E[X_n] = \rho^{n-1}, \quad n \geq 1.$$
Common Mistakes

- $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.
Common Mistakes

- $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.
  
  Let $X = 0, Y = 1$. 

$\sum_{n=0}^{\infty} a_n = \frac{1}{a}$. 

CS70 is difficult. 

I will do poorly on the final. 

Walrand is really weird. Probably!
Common Mistakes

- $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.

  Let $X = 0, Y = 1$. No: They are independent.
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  Let $X = 0, Y = 1$. **No**: They are independent.

  Let

  $X(1) = -1, X(2) = 0, X(1) = 1, Y(1) = 0, Y(2) = 1, Y(3) = 0$. 

**Common Mistakes**

- $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.
  - Let $X = 0, Y = 1$. **No**: They are independent.
  - Let $X(1) = -1, X(2) = 0, X(1) = 1, Y(1) = 0, Y(2) = 1, Y(3) = 0$.
- $3 \times 3.5 = 12.5$. 
Common Mistakes

- ▶ $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.
  
  Let $X = 0, Y = 1$. **No:** They are independent.

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  $X(1) = -1, X(2) = 0, X(1) = 1, Y(1) = 0, Y(2) = 1, Y(3) = 0$.

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- $3 \times 3.5 = 12.5$. No.

Common Mistakes

- Ω = \{1, 2, 3\}. Define X, Y with \(\text{cov}(X, Y) = 0\) and X, Y not independent.
  
  Let X = 0, Y = 1. No: They are independent.

  Let 
  \[ X(1) = -1, X(2) = 0, X(1) = 1, Y(1) = 0, Y(2) = 1, Y(3) = 0. \]

- \(3 \times 3.5 = 12.5\). No.

- \(E[X^2] = E[X]^2\). No.

- \(X = B(n, p) \implies \text{var}(X) = n^2 p(1 - p)\).
Common Mistakes

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