1. **Writing in propositional logic**

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

(a) The square of a nonzero integer is positive.

(b) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.

(d) For any two distinct real numbers, we can find a rational number in between them.

2. **Implication**

Which of the following implications are true? Give a counterexample for each false assertion.

(a) $\forall x, \forall y, P(x, y) \implies \forall y, \forall x, P(x, y)$.

(b) $\exists x, \exists y, P(x, y) \implies \exists y, \exists x, P(x, y)$.

(c) $\forall x, \exists y, P(x, y) \implies \exists y, \forall x, P(x, y)$.

(d) $\exists x, \forall y, P(x, y) \implies \forall y, \exists x, P(x, y)$.
3. **Perfect Square**

A *perfect square* is an integer $n$ of the form $n = m^2$ for some integer $m$. Prove that every odd perfect square is of the form $8k + 1$ for some integer $k$.

4. **Irrationals**

Prove that $2^{1/n}$ is not rational for any integer $n > 3$. [Hint : Fermat’s Last Theorem states that for all integers $n > 2$, there do not exist three positive integers $a, b, c$ that satisfy $a^n + b^n = c^n$.]

5. **Pigeonhole Principle**

Prove that if you put $n + 1$ apples into $n$ boxes, any way you like, then at least one box must contain at least 2 apples. This is known as the *pigeonhole principle*.

6. **Numbers of Friends**

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.