1. **Countability Basics**

   (a) Is \( f : \mathbb{N} \to \mathbb{N} \), defined by \( f(n) = n^2 \) an injection (one-to-one)? Briefly justify.

   (b) Is \( f : \mathbb{R} \to \mathbb{R} \), defined by \( f(x) = x^3 + 1 \) a surjection (onto)? Briefly justify.

2. **Count it!**

   For each of the following collections, determine and briefly explain whether it is finite, countably infinite (like the natural numbers), or uncountably infinite (like the reals):

   (a) The integers which divide 8.

   (b) The integers which 8 divides.

   (c) The functions from \( \mathbb{N} \to \mathbb{N} \).

   (d) Computer programs that halt.

   (e) Computer programs that always correctly tell if a program halts or not.

   (f) Numbers that are the roots of nonzero polynomials with integer coefficients.

   (g) The number of points in the unit square \([0, 1] \times [0, 1]\)

   (h) Computer programs that correctly return the product of their two integer arguments

3. **Countable and Uncountable.**

   (a) Give a bijection from the real number interval \((1, \infty)\) to the real number interval \((0, 1)\). (Notice the intervals are open.)

   (b) Given an \( n \times n \) matrix \( A \) where the diagonal consist of alternating 1’s and 0’s starting from 1, \( A[0, 0] = 1 \), describe a \( n \) length vector from \( \{0, 1\}^n \) that is not equal to a row in the matrix. (Hint: the all ones vector or the all zeros vector of length \( n \) could each be rows in the matrix.)

4. **Computability.**

   (a) The problem of determining whether a program halts in time \( 2^{n^2} \) on an input of size \( n \) is undecidable. (True or False.)

   (b) There is no computer program DEAD which takes a program \( P \), an input, \( x \), and a line number, \( n \), and determines whether the \( n \)th line of code is executed when the program \( P \) is run on the input \( x \). (True or False.)