70: Discrete Math and Probability Theory

Superpower!

What are your super powerful programs/processors doing?

Logic and Proofs!

Induction ≡ Recursion.

What can computers do?

Work with discrete objects.

Discrete Math = ⇒ immense application.

Computers learn and interact with the world?

E.g. machine learning, data analysis, robotics, ...

Probability!

See note 1, for more discussion.
70: Discrete Math and Probability Theory

Programming + Microprocessors
Programming + Microprocessors ≡ Superpower!
Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
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Logic and Proofs!
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What can computers do?
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  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?
  Work with discrete objects.
Programming + Microprocessors \equiv Superpower!

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    Induction \equiv Recursion.

What can computers do?
  Work with discrete objects.
Discrete Math
Programming + Microprocessors ≡ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction ≡ Recursion.

What can computers do?
  Work with discrete objects.
  Discrete Math ⇒ immense application.
Programming + Microprocessors ≡ Superpower!

What are your super powerful programs/processors doing?
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What can computers do?
   Work with discrete objects.
   Discrete Math ⇒ immense application.

Computers learn and interact with the world?
Programming + Microprocessors $\equiv$ Superpower!

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   Induction $\equiv$ Recursion.

What can computers do?
   Work with discrete objects.
   **Discrete Math** $\implies$ immense application.

Computers learn and interact with the world?
   E.g. machine learning, data analysis, robotics, ...
Programming + Microprocessors ≡ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction ≡ Recursion.

What can computers do?
  Work with discrete objects.
  **Discrete Math** ✶ly immense application.

Computers learn and interact with the world?
  E.g. machine learning, data analysis, robotics, ...
  **Probability!**
Programming + Microprocessors \(\equiv\) Superpower!

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   Logic and Proofs!
   Induction \(\equiv\) Recursion.

What can computers do?
   Work with discrete objects.
   **Discrete Math** \(\implies\) immense application.

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   E.g. machine learning, data analysis, robotics, ...
   **Probability**!

See note 1, for more discussion.
Instructor: Sanjit Seshia.
Instructors

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Taught: 149, 172, 144/244, 219C, EECS149.1x on edX, ...
I was born in Belgium(1) and came to Berkeley for my PhD. I have been teaching at UCB since 1982.

My wife and I live in Berkeley. We have two daughters (UC alumni – Go Bears!). We like to ski and play tennis (both poorly). We enjoy classical music and jazz.

My research interests include stochastic systems, networks and game theory.

(1)
Course Webpage: [http://www.eecs70.org/](http://www.eecs70.org/)
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Questions/Announcements ➞ piazza:
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Questions/Announcements ⇒ piazza: piazza.com/berkeley/fall2016/cs70
CS70: Lecture 1. Outline.

Today: Note 1.
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The language of proofs!
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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2+2 = 4 \]
\[ 2+2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
Jon Stewart is a good comedian
All evens \( > 2 \) are unique sums of 2 primes
\[ 4 + 5 \]
\[ x + x \]
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2+2 = 4 \]
\[ 2+2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
Jon Stewart is a good comedian
All evens > 2 are unique sums of 2 primes
\[ 4 + 5 \]
\[ x + x \]
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \quad \text{Proposition True} \]
\[ 2 + 2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
\[ \text{Jon Stewart is a good comedian} \]
\[ \text{All evens} > 2 \text{ are unique sums of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
Propositions: Statements that are true or false.

\( \sqrt{2} \) is irrational
2 + 2 = 4
2 + 2 = 3
826th digit of pi is 4
Jon Stewart is a good comedian
All evens \( > 2 \) are unique sums of 2 primes
4 + 5
x + x
Propositions: Statements that are true or false.

√2 is irrational
2+2 = 4
Proposition True
2+2 = 3
826th digit of pi is 4
Jon Stewart is a good comedian
Not a Proposition
All evens > 2 are unique sums of 2 primes
Proposition False
4 + 5
x + x
Not a Proposition.
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
\[ 2 + 2 = 3 \]
\[ \text{826th digit of pi is 4} \]
\[ \text{Jon Stewart is a good comedian} \]
\[ \text{All evens} > 2 \text{ are unique sums of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
Proposition: True

2 + 2 = 4
Proposition: True

2 + 2 = 3
Proposition: False

826th digit of pi is 4
Proposition: False

Jon Stewart is a good comedian
Not a Proposition

All evens \( > 2 \) are unique sums of 2 primes
Proposition: False

4 + 5
Not a Proposition

x + x
Not a Proposition
Propositions: Statements that are true or false.

\[
\begin{align*}
\sqrt{2} & \text{ is irrational} & \text{Proposition} & \text{True} \\
2 + 2 & = 4 & \text{Proposition} & \text{True} \\
2 + 2 & = 3 & \text{Proposition} & \text{False} \\
826\text{th digit of pi} & \text{ is 4} & \text{Proposition} & \text{False} \\
\text{Jon Stewart} & \text{ is a good comedian} & \text{Proposition} & \text{} \\
\text{All evens} > 2 & \text{ are unique sums of 2 primes} & \text{Proposition} & \text{} \\
4 + 5 & \text{} & \text{Proposition} & \text{} \\
x + x & \text{} & \text{Proposition} & \text{False} \\
\end{align*}
\]
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational
- $2+2 = 4$
- $2+2 = 3$
- 826th digit of pi is 4
- Jon Stewart is a good comedian
- All evens $> 2$ are unique sums of 2 primes
- $4 + 5$
- $x + x$

True, True, False, False, False

Again: "value" of a proposition is...

True or False
Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
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Jon Stewart is a good comedian
All evens $> 2$ are unique sums of 2 primes
4 + 5
$x + x$
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
Proposition True

\[ 2+2 = 4 \]
Proposition True

\[ 2+2 = 3 \]
Proposition False

826th digit of pi is 4
Proposition False

Jon Stewart is a good comedian
Not a Proposition

All evens \( > 2 \) are unique sums of 2 primes
Proposition

\[ 4 + 5 \]

\[ x + x \]
Propositions: Statements that are true or false.

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\[ 2 + 2 = 4 \quad \text{Proposition} \quad \text{True} \]
\[ 2 + 2 = 3 \quad \text{Proposition} \quad \text{False} \]
\[ \text{826th digit of pi is 4} \quad \text{Proposition} \quad \text{False} \]
\[ \text{Jon Stewart is a good comedian} \quad \text{Not a Proposition} \]
\[ \text{All evens } > 2 \text{ are unique sums of 2 primes} \quad \text{Proposition} \quad \text{False} \]
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Propositions: Statements that are true or false.

\[
\begin{align*}
\sqrt{2} & \text{ is irrational} & \text{Proposition} & \text{True} \\
2+2 &= 4 & \text{Proposition} & \text{True} \\
2+2 &= 3 & \text{Proposition} & \text{False} \\
826\text{th digit of pi} & \text{ is 4} & \text{Proposition} & \text{False} \\
\text{Jon Stewart is a good comedian} & & \text{Not a Proposition} & \\
\text{All evens} > 2 & \text{ are unique sums of 2 primes} & \text{Proposition} & \text{False} \\
4+5 & & \text{Not a Proposition} & \\
x + x & & \text{Not a Proposition} & \\
\end{align*}
\]
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Proposition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>Proposition</td>
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</tr>
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<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>All evens $&gt; 2$ are unique sums of 2 primes</td>
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</table>

Again: “value” of a proposition is ...
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
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All evens \( > 2 \) are unique sums of 2 primes
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Again: “value” of a proposition is ... True or False
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is **True** when both $P$ and $Q$ are **True**.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

\( P \land Q \) is \textbf{True} when both \( P \) and \( Q \) are \textbf{True}. Else \textbf{False}.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$2 + 2 = 3$" – a proposition that is False.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True when both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

“$P \lor Q$” is True when at least one $P$ or $Q$ is True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

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Conjunction ("and"): $P \land Q$

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Disjunction ("or"): $P \lor Q$

“$P \lor Q$” is True when at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$
Propositional Forms.

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Negation (“not”): \( \neg P \)

“\( \neg P \)” is True when \( P \) is False.
Propositional Forms.

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Examples:
Propositional Forms.

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Negation (“not”): \( \neg P \)

“\( \neg P \)” is True when \( P \) is False. Else False.

Examples:

\( \neg \ “(2 + 2 = 4)” \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.

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"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg \ \"(2 + 2 = 4)\" \) – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is True when both \( P \) and \( Q \) are True . Else False .

Disjunction (“or”): \( P \lor Q \)

“\( P \lor Q \)” is True when at least one \( P \) or \( Q \) is True . Else False .

Negation (“not”): \( \neg P \)

“\( \neg P \)” is True when \( P \) is False . Else False .

Examples:

\( \neg \) “\( 2 + 2 = 4 \)” – a proposition that is ... False

“\( 2 + 2 = 3 \)” \( \land \) “\( 2 + 2 = 4 \)” – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True when both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

“$P \lor Q$” is True when at least one $P$ or $Q$ is True. Else False.

Negation (“not”): $\neg P$

“$\neg P$” is True when $P$ is False. Else False.

Examples:

$\neg \text{“(2 + 2 = 4)”}$ – a proposition that is ... False

“$2 + 2 = 3$” $\land$ “$2 + 2 = 4$” – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True when at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is ... False

"$2 + 2 = 3$" $\lor$ "$2 + 2 = 4$" – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

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"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

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Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"2 + 2 = 3" \land "2 + 2 = 4" – a proposition that is ... False

"2 + 2 = 3" \lor "2 + 2 = 4" – a proposition that is ... True
Propositional Forms.

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"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"2 + 2 = 3" \land "2 + 2 = 4" – a proposition that is ... False

"2 + 2 = 3" \lor "2 + 2 = 4" – a proposition that is ... True
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
Propositional Forms: quick check!

\[ P = "\sqrt{2} \text{ is rational}" \]
\[ Q = "826th digit of pi is 2" \]
Propositional Forms: quick check!

\[ P = \text{"} \sqrt{2} \text{ is rational} \]
\[ Q = \text{"} 826\text{th digit of pi is 2} \]
Propositional Forms: quick check!

$P = \text{“} \sqrt{2} \text{ is rational”}$

$Q = \text{“} 826\text{th digit of pi is 2”}$

$P$ is ...

$P \land Q$ ...

$P \lor Q$ ...

$\neg P$ ...


Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2} \text{”} \]

\[ P \text{ is ...False} . \]
Propositional Forms: quick check!

$P = \text{"} \sqrt{2} \text{ is rational}\text{"}$
$Q = \text{"} 826\text{th digit of pi is 2}\text{"}$

$P$ is ... False.
$Q$ is ...
Propositional Forms: quick check!

\[ P = \text{“\(\sqrt{2}\) is rational”} \]
\[ Q = \text{“826th digit of pi is 2”} \]

\[ P \text{ is } \text{False} . \]
\[ Q \text{ is } \text{True} . \]
Propositional Forms: quick check!

\[ P = \text{"} \sqrt{2} \text{ is rational}\]
\[ Q = \text{"} 826\text{th digit of pi is 2}\]

\[ P \text{ is ...} \text{False} \ . \]
\[ Q \text{ is ...} \text{True} \ . \]

\[ P \land Q \ldots \]
Propositional Forms: quick check!

\( P = \text{“} \sqrt{2} \text{ is rational”} \)
\( Q = \text{“} 826 \text{th digit of pi is 2”} \)

\( P \) is ... False.
\( Q \) is ... True.

\( P \land Q \) ... False
Propositional Forms: quick check!

$P = "\sqrt{2} \text{ is rational}"$
$Q = "826th digit of pi is 2"

$P$ is ... False.
$Q$ is ... True.

$P \land Q$ ... False
$P \lor Q$ ...
Propositional Forms: quick check!

\[ P = "\sqrt{2} \text{ is rational}" \]
\[ Q = "826th digit of pi is 2" \]

\[ P \] is \textbf{False}.
\[ Q \] is \textbf{True}.

\[ P \land Q \] \textbf{False}

\[ P \lor Q \] \textbf{True}


Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational} \text{”} \]
\[ Q = \text{“} 826 \text{th digit of pi is 2} \text{”} \]

\[ P \text{ is ...False .} \]
\[ Q \text{ is ...True .} \]

\[ P \land Q \ ... \text{ False} \]
\[ P \lor Q \ ... \text{ True} \]
\[ \neg P \ ... \]
Propositional Forms: quick check!

\[ P = \text{“}\sqrt{2} \text{ is rational”} \]
\[ Q = \text{“}826\text{th digit of pi is 2”} \]

\[ P \text{ is ...} \text{False} \]
\[ Q \text{ is ...} \text{True} \]

\[ P \land Q \text{ ... False} \]
\[ P \lor Q \text{ ... True} \]
\[ \neg P \text{ ... True} \]
Propositional Forms: quick check!

\( P = "\sqrt{2} \) is rational"
\( Q = "826th digit of pi is 2" \)

\( P \) is ... \text{False} .
\( Q \) is ... \text{True} .

\( P \land Q \) ... \text{False}
\( P \lor Q \) ... \text{True}
\( \neg P \) ... \text{True}
Put them together..

Propositions:

$P_1$ - Person 1 rides the bus.
Put them together..

Propositions:

\[ P_1 \] - Person 1 rides the bus.
\[ P_2 \] - Person 2 rides the bus.
Put them together..

Propositions:

\[ P_1 \] - Person 1 rides the bus.
\[ P_2 \] - Person 2 rides the bus.

....
Put them together..

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\[ P_1 \] - Person 1 rides the bus.
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Suppose we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn’t.
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Propositional Form:
\[
\neg (((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor (((P_2 \lor P_3) \land (P_4 \lor \neg P_5))))
\]
Put them together..

Propositions:
P_1 - Person 1 rides the bus.
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\neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))

Who can ride the bus?
Propositions:
$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.
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Who can ride the bus?
What combinations of people can ride the bus?
Put them together..

Propositions:
P_1 - Person 1 rides the bus.
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Suppose we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn’t.

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Who can ride the bus?
What combinations of people can ride the bus?
This seems ...
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Who can ride the bus?
What combinations of people can ride the bus?

This seems ... *complicated*.

We need a way to keep track!
Truth Tables for Propositional Forms.

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One use for truth tables: Logical Equivalence of propositional forms!

Example:

\[ \neg (P \land Q) \] logically equivalent to \[ \neg P \lor \neg Q \]

...because the two propositional forms have the same...

...Truth Table!

DeMorgan's Law for Negation: distribute and flip!

\[ \neg (P \land Q) \equiv \neg P \lor \neg Q \]

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One use for truth tables: Logical Equivalence of propositional forms!

Example:

$\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$...because the two propositional forms have the same...

DeMorgan's Law's for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

$\neg (P \lor Q) \equiv \neg P \land \neg Q$
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DeMorgan's Law's for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$  
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DeMorgan's Law's for Negation: distribute and flip!

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One use for truth tables: Logical Equivalence of propositional forms!
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One use for truth tables: Logical Equivalence of propositional forms!
Example: \( \neg(P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \)
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One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$
...because the two propositional forms have the same...
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One use for truth tables: Logical Equivalence of propositional forms! Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$ ...because the two propositional forms have the same... ....Truth Table!

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DeMorgan's Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$

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Truth Tables for Propositional Forms.

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Truth Tables for Propositional Forms.

\[
\begin{array}{c|c|c|}
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\quad \quad \quad \quad \quad \quad
\begin{array}{c|c|c|}
P & Q & P \lor Q \\
\hline
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg(P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \)

...because the two propositional forms have the same...

...Truth Table!

\[
\begin{array}{c|c|c|c|}
P & Q & \neg(P \land Q) & \neg P \lor \neg Q \\
\hline
T & T & F & F \\
T & F & F & F \\
F & T & F & F \\
F & F & T & F \\
\end{array}
\]

DeMorgan's Law's for Negation: distribute and flip!
Truth Tables for Propositional Forms.

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DeMorgan’s Law’s for Negation: distribute and flip!

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<th>$\neg(P \land Q)$</th>
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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$
$\neg(P \lor Q) \equiv \neg P \land \neg Q$
Implication.

\[ P \implies Q \] interpreted as

True Statements: \( P, P \implies Q \).

Conclude: \( Q \) is true.

Example:

Statement: If you stand in the rain, then you'll get wet.

\( P \) = "you stand in the rain"
\( Q \) = "you will get wet"

Statement: "Stand in the rain"

Can conclude: "you'll get wet."
Implication.

$P \implies Q$ interpreted as
If $P$, then $Q$. 

True Statements: $P \implies P = \Rightarrow Q$. 

Conclude: $Q$ is true.

Example: Statement: If you stand in the rain, then you'll get wet.

$P = \text{"you stand in the rain"}$

$Q = \text{"you will get wet"}$

Statement: "Stand in the rain"
Can conclude: "you'll get wet."
Implication.

\[ P \implies Q \text{ interpreted as } \]
\[ \text{If } P, \text{ then } Q. \]
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).

Example:

Statement: If you stand in the rain, then you'll get wet.

\( P = \text{"you stand in the rain"}, Q = \text{"you will get wet"} \)

Can conclude: "you'll get wet."
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.
Implication.

\[ P \implies Q \text{ interpreted as } \]

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Example:
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, \ P \implies Q \).
Conclude: \( Q \) is true.

Example: Statement: If you stand in the rain, then you’ll get wet.
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Example: Statement: If you stand in the rain, then you’ll get wet.
\[ P = \text{“you stand in the rain”} \]
Implication.

$P \implies Q$ interpreted as

If $P$, then $Q$.

True Statements: $P, P \implies Q$.
Conclude: $Q$ is true.

Example: Statement: If you stand in the rain, then you’ll get wet.

$P = “you stand in the rain”$
$Q = “you will get wet”$
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Example: Statement: If you stand in the rain, then you’ll get wet.

\[ P = \text{“you stand in the rain”} \]
\[ Q = \text{“you will get wet”} \]
Statement: “Stand in the rain”
Implication.

\[ P \implies Q \text{ interpreted as} \]
\[ \text{If } P, \text{ then } Q. \]

True Statements: \( P, P \implies Q. \)
Conclude: \( Q \) is true.

Example: Statement: If you stand in the rain, then you’ll get wet.
\[ P = \text{“you stand in the rain”} \]
\[ Q = \text{“you will get wet”} \]
Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”
Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)” only is False if \( P \) is True and \( Q \) is False.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing

P False means
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False .

False implies nothing

$P$ False means $Q$ can be True or False
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means $Q$ can be True or False
Anything implies true.

Be careful out there!

Some Fun: use propositional formulas to describe implication?

$((P \implies Q) \land P) \implies Q$. 
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing

$P$ False means $Q$ can be True or False

Anything implies true.

$P$ can be True or False when
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is \textbf{False} if $P$ is \textbf{True} and $Q$ is \textbf{False}.

False implies nothing
P False means $Q$ can be \textbf{True} or \textbf{False}
Anything implies true.
P can be \textbf{True} or \textbf{False} when $Q$ is \textbf{True}

Be careful out there!

Some Fun: use propositional formulas to describe implication?
\[(P \implies Q) \land P \implies Q\]
The statement “$P \implies Q$”
only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing
$P$ **False** means $Q$ can be **True** or **False**
Anything implies true.
$P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
The statement \( P \implies Q \)

only is \textbf{False} if \( P \) is \textbf{True} and \( Q \) is \textbf{False}.

False implies nothing
P \textbf{False} means \( Q \) can be \textbf{True} or \textbf{False}
Anything implies true.
P can be \textbf{True} or \textbf{False} when \( Q \) is \textbf{True}

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?
The statement “$P \implies Q$” only is False if $P$ is True and $Q$ is False.

False implies nothing

$P$ False means $Q$ can be True or False

Anything implies true.

$P$ can be True or False when $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?

Not necessarily.
Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)” only is False if \( P \) is True and \( Q \) is False.

False implies nothing.
\( P \) False means \( Q \) can be True or False.
Anything implies true.
\( P \) can be True or False when \( Q \) is True.

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?
Not necessarily.

\( P \implies Q \) and \( Q \) are True does not mean \( P \) is True.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

*False implies nothing*

$P$ False means $Q$ can be **True** or **False**

*Anything implies true.*

$P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?

Not necessarily.

$P \implies Q$ and $Q$ are **True** does not mean $P$ is **True**

Instead we have:
Non-Consequences/consequences of Implication

The statement “$P \Rightarrow Q$”
only is **False** if $P$ is **True** and $Q$ is **False**.

**False implies nothing**

$P$ **False** means $Q$ can be **True** or **False**

**Anything implies true.**

$P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?

Not necessarily.

$P \Rightarrow Q$ and $Q$ are **True** does not mean $P$ is **True**

Instead we have:

$P \Rightarrow Q$ and $P$ are **True** does **mean** $Q$ is **True**.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$” only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False when $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant polluted river?
Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Instead we have:
$P \implies Q$ and $P$ are True does mean $Q$ is True.

Be careful out there!
The statement “$P \implies Q$”

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Some Fun: use propositional formulas to describe implication?
Non-Consequences/consequences of Implication

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Not necessarily.

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Instead we have:

$P \implies Q$ and $P$ are True does mean $Q$ is True.

Be careful out there!

Some Fun: use propositional formulas to describe implication?

$((P \implies Q) \land P) \implies Q$. 
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
Implication and English.

$P \implies Q$

- If $P$, then $Q$.
- $Q$ if $P$. 
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
- \( P \) only if \( Q \).
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
- \( P \) only if \( Q \).
- \( P \) is sufficient for \( Q \).
- \( Q \) is necessary for \( P \).
Implication and English.

\[ P \implies Q \]

- If $P$, then $Q$.
- $Q$ if $P$.
- $P$ only if $Q$.
- $P$ is sufficient for $Q$.
- $Q$ is necessary for $P$. 
Truth Table: implication.

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<th>$P$</th>
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These two propositional forms are logically equivalent!
Truth Table: implication.

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$\neg P \lor Q \equiv P \implies Q.$
Truth Table: implication.

\[
\begin{array}{ccc}
P & Q & P \implies Q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

\[
\begin{array}{ccc}
P & Q & \neg P \lor Q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

\[\neg P \lor Q \equiv P \implies Q.\]

These two propositional forms are logically equivalent!
Contrapositive, Converse

- Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

- Converse of $P \implies Q$ is $Q \implies P$.
  - If fish die, the plant pollutes.

If you stand in the rain, you get wet.
- If you did not stand in the rain, you did not get wet.

- If you did not get wet, you did not stand in the rain.

If you stand in the rain, you get wet.
- If you did not stand in the rain, you did not get wet.

- If you did not get wet, you did not stand in the rain.

Definition:
If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P$.

(Logically Equivalent: $\iff$.)
Contraposition, Converse

- Contraposition of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
  (contrapositive)
Contrapositive, Converse

Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

- If the plant pollutes, fish die.
- If the fish don’t die, the plant does not pollute. (contrapositive)

- If you stand in the rain, you get wet.
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.

- Converse of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

- Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$.
  (Logically Equivalent: $\iff$.)
Contrapositive, Converse

- Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.  
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.  
    (not contrapositive!)
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.

- Converse of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

- Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$. (Logically Equivalent: $\iff$).
Contrapositive, Converse

- Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).
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Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \].

No. They have a free variable. We call them predicates, e.g.,

\[ Q(x) = "x \text{ is even}" \]

Same as boolean valued functions from 61A or 61AS!

\[ P(n) = "\sum_{i=1}^{n} i = \frac{n(n+1)}{2}" \]

\[ R(x) = "x > 2" \]

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Next: Statements about boolean valued functions!!
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Next: Statements about boolean valued functions!!
There exists quantifier:

\( \exists x \in S \) \( P(x) \) means "\( P(x) \) is true for some \( x \) in \( S \)".

Wait!

What is \( S \)?

\( S \) is the universe: "the type of \( x \)".

Universe examples include:

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\} \) (integers).
- \( \mathbb{Z}^+ \) (positive integers).

See note 0 for more!
Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "$P(x)$ is true for some $x$ in $S$"
Quantifiers.

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means "$P(x)" is true for some $x$ in $S$"

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Universe examples include:

- $\mathbb{N} = \{0, 1, 2, ..., \}$ (natural numbers).
- $\mathbb{Z} = \{..., -1, 0, 1, 2, ..., \}$ (integers).
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Quantifiers.

There exists quantifier:

$\exists x \in S (P(x))$ means "$P(x)$ is true for some $x$ in $S$"

For example:

$\exists x \in \mathbb{N} (x = x^2)$

Equivalent to "$0 = 0 \lor 1 = 1 \lor 2 = 4 \lor \ldots$"

Much shorter to use a quantifier!

For all quantifier:

$\forall x \in S (P(x))$. means "For all $x$ in $S$ $P(x)$ is True."

Examples:

"Adding 1 makes a bigger number."

$\forall x \in \mathbb{N} (x + 1 > x)$

"the square of a number is always non-negative"

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Quantifiers..

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Equivalent to "(0 = 0)"
Quantifiers..

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Equivalent to “\(0 = 0\) ∨ (1 = 1)"
Quantifiers.

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For example:
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Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4)\)"
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\((\forall x \in N) (x + 1 > x)\)

"the square of a number is always non-negative"

\((\forall x \in N)(x^2 \geq 0)\)
Quantifiers are not commutative.

- Consider this English statement: "there is a natural number that is the square of every natural number", i.e the square of every natural number is the same number!
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\[ (\exists y \in \mathbb{N}) \ (\forall x \in \mathbb{N}) \ (y = x^2) \]
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\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False
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\[ (\exists y \in N) \ (\forall x \in N) \ (y = x^2) \quad \text{False} \]

- Consider this one: "the square of every natural number is a natural number"...
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\[(\forall x \in N)(\exists y \in N) (y = x^2)\] True
Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an \(x\) in \(S\) where \(P(x)\) does not hold.

What we do in this course! We consider claims.

Claim: \((\forall x) P(x)\)

"For all inputs \(x\) the program works."

For False, find \(x\), where \(\neg P(x)\).

Counterexample.

Bad input. Case that illustrates bug.

For True: prove claim.

Next lectures...
Quantifiers....negation...DeMorgan again.

Consider

\[ \neg (\forall x \in S)(P(x)), \]

By DeMorgan’s law,
Quantifiers...negation...DeMorgan again.

Consider

\[ \neg(\forall x \in S)(P(x)), \]

By DeMorgan’s law,

\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]
Quantifiers....negation...DeMorgan again.

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\[\neg (\forall x \in S) (P(x)),\]

By DeMorgan’s law,

\[\neg (\forall x \in S) (P(x)) \iff \exists (x \in S) (\neg P(x)).\]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
Quantifiers....negation...DeMorgan again.

Consider

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English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

What we do in this course! We consider claims.
Consider
\[ \neg (\forall x \in S)(P(x)), \]

By DeMorgan’s law,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

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**Claim:** \( (\forall x) P(x) \)
Quantifiers....negation...DeMorgan again.

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Claim: \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”

For False, find \( x \), where \( \neg P(x) \).
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Counterexample.

Bad input.
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Negation of exists.

Consider

$\neg (\exists x \in S) (P(x))$

Equivalent to:

$\neg (\exists x \in S) (P(x)) \iff \forall (x \in S) \neg P(x)$.

English: means that for all $x$ in $S$, $P(x)$ does not hold.
Negation of exists.

Consider

\[ \neg (\exists x \in S)(P(x)) \]

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$$\neg(\exists x \in S)(P(x))$$

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Equivalent to:

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English: means that for all x in S, P(x) does not hold.
Which Theorem?

Theorem: $\forall n \in N \ (n \geq 3 \implies \neg (\exists a, b, c \in N \ a^n + b^n = c^n))$
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Which Theorem?

Fermat’s Last Theorem!
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Remember Right-Angled Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...
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1637: Proof doesn’t fit in the margins.
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DeMorgan Restatement:
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Theorem: $\neg(\exists n \in N \exists a, b, c \in N \ (n \geq 3 \land a^n + b^n = c^n))$
Summary.

Propositions are statements that are true or false.
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Propositions are statements that are true or false.
Propositional forms use \( \land, \lor, \neg \).

DeMorgan's Laws: "Flip and Distribute negation"

\[ \neg (P \lor Q) \iff \neg P \land \neg Q \]
\[ \neg \forall x P(x) \iff \exists x \neg P(x) \]

Next Time: proofs!
Summary.

Propositions are statements that are true or false. Propositional forms use \( \land, \lor, \neg \).

The meaning of a propositional form is given by its truth table.
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Implication: $P \implies Q \iff \neg P \lor Q$. 

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Converse: $Q \implies P$

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Predicates: Statements with “free” variables.

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Now can state theorems!
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Propositions are statements that are true or false.

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Next Time: proofs!