Barber paradox.

Created by logician Bertrand Russell.
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Village with just 1 barber, all men clean-shaven.
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Village with just 1 barber, all men clean-shaven. Barber announces:
Barber paradox.

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Village with just 1 barber, all men clean-shaven. Barber announces:
“\textit{I shave all and only those men who do not shave themselves.”}
Barber paradox.

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Village with just 1 barber, all men clean-shaven. Barber announces:
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Who shaves the barber?
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Who shaves the barber?

Case 1: It’s the barber.
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Who shaves the barber?

Case 1: It’s the barber.
Case 2: Somebody else.
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Case 1: It’s the barber. 
Case 2: Somebody else. 

Cannot answer that question in either case!
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Case 1: It’s the barber.
Case 2: Somebody else.

Cannot answer that question in either case! Paradox!!!
Russell’s Paradox.

Naive Set Theory: Any definable collection is a set.
Russell’s Paradox.

Naive Set Theory: Any definable collection is a set.

\[ \exists y \forall x \ (x \in y \iff P(x)) \quad (1) \]
Russell’s Paradox.

Naive Set Theory: Any definable collection is a set.

\[ \exists y \ \forall x \ (x \in y \iff P(x)) \]  

(1)

\(y\) is the set of elements that satisfies the proposition \(P(x)\).
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\( P(x) = x \notin x. \)
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Take \( x = y \).

\[ y \in y \iff y \notin y. \]

Oops!
Is this stuff actually useful?
Is this stuff actually useful?

Verify that my program is correct!

Determines if $P(I)$ halts or loops forever.

Notice: Need a computer...with the notion of a stored program!!!! (not an adding machine! not a person and an adding machine.)

Program is a text string. Text string can be an input to a program. Program can be an input to a program.
Is this stuff actually useful?

Verify that my program is correct!
Check that the compiler works!
Is this stuff actually useful?

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How about.. Check that the compiler terminates on a certain input.
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\textit{HALT}(P, I)
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\[ \text{HALT}(P, I) \]

\( P \) - program
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Text string can be an input to a program.
Program can be an input to a program.
Implementing HALT.

HALT \((P, I)\)

- \(P\) - program
- \(I\) - input.

Determines if \(P(I)\) (run on \(I\)) halts or loops forever.

Run \(P\) on \(I\) and check!

How long do you wait?
Implementing HALT.

\[ \text{HALT}(P, I) \]
\begin{align*}
P & \text{ - program} \\
I & \text{ - input.}
\end{align*}

Determines if \( P(I) \) (\( P \) run on \( I \)) halts or loops forever.
Implementing HALT.

\[ HALT(P, I) \]
\[ P - \text{ program} \]
\[ I - \text{ input.} \]

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Run \( P \) on \( I \) and check!
Implementing HALT.

HALT\((P, I)\)

- \(P\) - program
- \(I\) - input.

Determines if \(P(I)\) (\(P\) run on \(I\)) halts or loops forever.

Run \(P\) on \(I\) and check!

How long do you wait?
Halt does not exist.
Halt does not exist.

$\text{HALT}(P, I)$

Theorem: There is no program $\text{HALT}$.

Halt does not exist.

\[ \text{HALT}(P, I) \]

\( P \) - program
Halt does not exist.

\[ HALT(P, I) \]
\[ P \] - program
\[ I \] - input.
Halt does not exist.

\[ \text{HALT}(P, I) \]
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**Theorem:** There is no program HALT.
Halt does not exist.

\[ \text{HALT}(P, I) \]
\begin{itemize}
  \item \( P \) - program
  \item \( I \) - input.
\end{itemize}

Determines if \( P(I) \) (\( P \) run on \( I \)) halts or loops forever.

**Theorem:** There is no program HALT.

**Proof Idea:** Proof by contradiction, use self-reference.
Halt and Turing.

Proof:
Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$. 
**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

Turing($P$)
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

$Turing(P)$
1. If $HALT(P,P) =$“halts”, then go into an infinite loop.
Halt and Turing.

**Proof:** Assume there is a program $HALT(·,·)$. 

**Turing($P$)**
1. If $HALT(P,P) =$“halts”, then go into an infinite loop.
2. Otherwise, halt immediately.
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

**Turing(P)**
1. If $HALT(P,P) = \text{"halts"}$, then go into an infinite loop.
2. Otherwise, halt immediately.

Assumption: there is a program $HALT$. 

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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1. If $HALT(P, P) =$“halts”, then go into an infinite loop.
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Assumption: there is a program HALT.
There is text that “is” the program HALT.

Program HALT does not exist!
Halt and Turing.

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Can run Turing on Turing!
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Does $Turing(Turing)$ halt?
Halt and Turing.

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$Turing(Turing)$ halts
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Does $Turing(Turing)$ halt?

$Turing(Turing)$ halts
$\implies$ then $HALT(Turing, Turing) = \text{halts}$
Halt and Turing.

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1. If $HALT(P, P) = \text{"halts"}$, then go into an infinite loop.
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Does Turing(Turing) halt?

Turing(Turing) halts
\[\implies \text{then } HALT(\text{Turing, Turing}) = \text{halts}\]
\[\implies \text{Turing}(\text{Turing}) \text{ loops forever.}\]
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot, \cdot)$.

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Turing(Turing) \text{ halts} \\
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$$

$Turing(Turing) \text{ loops forever}$
Halt and Turing.

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1. If $HALT(P,P) =$“halts”, then go into an infinite loop.
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Assumption: there is a program $HALT$. There is text that “is” the program $HALT$. There is text that is the program $Turing$. Can run $Turing$ on $Turing$!

Does $Turing(Turing)$ halt?

$Turing(Turing)$ halts
   $\implies$ then $HALT(Turing, Turing) =$ halts
   $\implies$ $Turing(Turing)$ loops forever.

$Turing(Turing)$ loops forever
   $\implies$ then $HALT(Turing, Turing) \neq$ halts
Halt and Turing.

**Proof:** Assume there is a program \( HALT(\cdot,\cdot) \).

\( \text{Turing}(P) \)
1. If \( HALT(P,P) = \text{"halts"} \), then go into an infinite loop.
2. Otherwise, halt immediately.

Assumption: there is a program HALT.
There is text that “is” the program HALT.
There is text that is the program Turing.
Can run Turing on Turing!

Does \( \text{Turing}(\text{Turing}) \) halt?

\( \text{Turing}(\text{Turing}) \) halts
\[ \implies \text{then } HALT(\text{Turing}, \text{Turing}) = \text{halts} \]
\[ \implies \text{Turing}(\text{Turing}) \text{ loops forever.} \]

\( \text{Turing}(\text{Turing}) \) loops forever
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1. If $HALT(P,P) =$“halts”, then go into an infinite loop.
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Contradiction.
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**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

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\[ \implies \text{Turing}(Turing) \text{ halts.} \]

Contradiction. Program $HALT$ does not exist!
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

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Another view of proof: diagonalization.

Any program is a fixed length string.
Another view of proof: diagonalization.

Any program is a fixed length string. Fixed length strings are enumerable.
Another view of proof: diagonalization.

Any program is a fixed length string.  
Fixed length strings are enumerable.  
Program halts or not any input, which is a string.
Another view of proof: diagonalization.

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Halt(\(P\), \(P\)) - diagonal.
Turing - is not Halt.
and is different from every \(P_i\) on the diagonal.
Turing is not on list.

Turing is not a program.
Turing can be constructed from Halt.
Halt does not exist!
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</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$P_3$</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
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</table>

Halt($P,P$) - diagonal.
Turing - is not Halt.
and is different from every $P_i$ on the diagonal.
Another view of proof: diagonalization.

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Turing is not on list.
Another view of proof: diagonalization.

Any program is a fixed length string. Fixed length strings are enumerable. Program halts or not any input, which is a string.

\[
\begin{array}{c|cccc}
P & P_1 & P_2 & P_3 & \ldots \\
\hline
P_1 & H & H & L & \ldots \\
P_2 & L & L & H & \ldots \\
P_3 & H & H & H & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
\end{array}
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Turing machine.

- An (infinite) tape with characters
- Be in a state, and read a character
- Move left, right, and/or write a character.

Universal Turing machine
- An interpreter program for a Turing machine
- Where the tape could be a description of a...

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Church, Gödel and Turing.

Church proved an equivalent theorem. (Previously.)
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Used $\lambda$ calculus....

Gödel: Incompleteness theorem. Any formal system either is inconsistent or incomplete.
Inconsistent: A false sentence can be proven.
Incomplete: There is no proof for some sentence in the system.
Along the way: “built” computers out of arithmetic.
Showed that every mathematical statement corresponds to a natural number!!!
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Summary: computability.

Computer Programs are interesting objects.
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Computer Programs are interesting objects.
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Computer Programs cannot completely "understand" computer programs.

Example: no computer program can tell if any other computer program HALTS.

Proof Idea: Diagonalization.
Program: Turing (or DIAGONAL) takes P.
Assume there is HALT.
DIAGONAL flips answer.
Loops if P halts, halts if P loops.

What does Turing do on turing?

Doesn't loop or HALT.
HALT does not exist!

More on this topic in CS 172.

Computation is a lens for other action in the world.
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