Probability Review
Probability Review
1. True or False

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1. True or False
2. Some Key Results
CS70: Jean Walrand: Review of Probability.

1. True or False
2. Some Key Results
3. Quiz 1: G
Probability Review

1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
Probability Review

1. True or False
2. Some Key Results
3. Quiz 1: G ($\approx 40\%$)
4. Quiz 2: PG
1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
4. Quiz 2: PG (≈ 40%)
Probability Review

1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
4. Quiz 2: PG (≈ 40%)
5. Quiz 3: R
Probability Review

1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
4. Quiz 2: PG (≈ 40%)
5. Quiz 3: R (≈ 20%)
 Probability Review

1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
4. Quiz 2: PG (≈ 40%)
5. Quiz 3: R (≈ 20%)
6. Common Mistakes
1. True or False
2. Some Key Results
3. Quiz 1: G (≈ 40%)
4. Quiz 2: PG (≈ 40%)
5. Quiz 3: R (≈ 20%)
6. Common Mistakes
True or False

- $\Omega$ and $A$ are independent.
True or False

- $\Omega$ and $A$ are independent. True
True or False

- Ω and A are independent.  True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$.  True
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
True or False

- Ω and A are independent. True
- \( Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B] \). True
- \( Pr[A \setminus B] \geq Pr[A] - Pr[B] \). True
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
True or False

- $\Omega$ and $A$ are independent.  True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1 + \cdots + X_n}{n}\right) = var(X_1)$. False:
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1+\cdots+X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}$
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1 + \ldots + X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$.
True or False

- \( \Omega \) and \( A \) are independent. True
- \( Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B] \). True
- \( Pr[A \setminus B] \geq Pr[A] - Pr[B] \). True
- \( X_1, \ldots, X_n \) i.i.d. \( \implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1) \). False: \( \times \frac{1}{n} \)
- \( Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \). True
True or False

- \( \Omega \) and \( A \) are independent. **True**
- \( \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \). **True**
- \( \Pr[A \setminus B] \geq \Pr[A] - \Pr[B] \). **True**
- \( X_1, \ldots, X_n \) i.i.d. \( \Rightarrow \) \( \text{var}\left( \frac{X_1 + \cdots + X_n}{n} \right) = \text{var}(X_1) \). **False:** \( \times \frac{1}{n} \)
- \( \Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \). **True**
- \( X_1, \ldots, X_n \) i.i.d. \( \Rightarrow \) \( \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \rightarrow \mathcal{N}(0,1) \).
True or False

- $\Omega$ and $A$ are independent. True
- $\Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B]$. True
- $\Pr[A \setminus B] \geq \Pr[A] - \Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}$
- $\Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \to \mathcal{N}(0,1)$. False: $\sqrt{n}$
True or False

- True or False
- \( \Omega \) and \( A \) are independent. True
- \( \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \). True
- \( \Pr[A \setminus B] \geq \Pr[A] - \Pr[B] \). True
- \( X_1, \ldots, X_n \) i.i.d. \( \implies \var\left(\frac{X_1 + \cdots + X_n}{n}\right) = \var(X_1) \). False: \( \times \frac{1}{n} \)
- \( \Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \). True
- \( X_1, \ldots, X_n \) i.i.d. \( \implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \to \mathcal{N}(0,1) \). False: \( \sqrt{n} \)
- \( X = Expo(\lambda) \implies \Pr[X > 5 | X > 3] = \Pr[X > 2] \).
True or False

- **True or False**
  - $\Omega$ and $A$ are independent. True
  - $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
  - $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
  - $X_1, \ldots, X_n$ i.i.d. $\implies \text{var}(\frac{X_1 + \cdots + X_n}{n}) = \text{var}(X_1)$. False: $\times \frac{1}{n}$
  - $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
  - $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \xrightarrow{d} N(0,1)$. False: $\sqrt{n}$
  - $X = \text{Expo}(\lambda) \implies Pr[X > 5 | X > 3] = Pr[X > 2]$. True:
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies var(\frac{X_1+\ldots+X_n}{n}) = var(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1+\ldots+X_n-nE[X_1]}{n\sigma(X_1)} \rightarrow \mathcal{N}(0,1)$. False: $\sqrt{n}$
- $X = \text{Expo}(\lambda) \implies Pr[X > 5|X > 3] = Pr[X > 2]$. True:
  $$\frac{\exp\{-\lambda 5\}}{\exp\{-\lambda 3\}} = \exp\{-\lambda 2\}.$$
Correct or not?
Correct or not?

When $n \gg 1$, one has
When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{n}, A_n + 2\sigma \frac{1}{n}] = 95\%$-CI for $\mu$.  

If $0.3 < \sigma < 3$, then $[A_n - 6.1 \sqrt{n}, A_n + 6.1 \sqrt{n}] = 95\%$-CI for $\mu$. 

Yes
Correct or not?

When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
Correct or not?

When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
Correct or not?

When \( n \gg 1 \), one has

- \( [A_n - 2\sigma \frac{1}{n}, A_n + 2\sigma \frac{1}{n}] \) = 95\%-CI for \( \mu \). No
- \( [A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] \) = 95\%-CI for \( \mu \). Yes
Correct or not?

When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{n}, A_n + 2\sigma \frac{1}{n}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
- If $0.3 < \sigma < 3$, then
  $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
Correct or not?

When $n \gg 1$, one has

- $[A_n - 2\sigma_n^{1/n}, A_n + 2\sigma_n^{1/n}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma_n^{1/n}, A_n + 2\sigma_n^{1/n}] = 95\%$-CI for $\mu$. Yes
- If $0.3 < \sigma < 3$, then $[A_n - 0.6\sigma_n^{1/n}, A_n + 0.6\sigma_n^{1/n}] = 95\%$-CI for $\mu$. No
Correct or not?

When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
- If $0.3 < \sigma < 3$, then
  $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- If $0.3 < \sigma < 3$, then
  $[A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
When $n \gg 1$, one has

- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- $[A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
- If $0.3 < \sigma < 3$, then $[A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. No
- If $0.3 < \sigma < 3$, then $[A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%$-CI for $\mu$. Yes
When \( n \gg 1 \), one has

- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu\). No
- \([A_n - 2\sigma \frac{1}{\sqrt{n}}, A_n + 2\sigma \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu\). Yes
- If \( 0.3 < \sigma < 3 \), then
  \([A_n - 0.6 \frac{1}{\sqrt{n}}, A_n + 0.6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu\). No
- If \( 0.3 < \sigma < 3 \), then
  \([A_n - 6 \frac{1}{\sqrt{n}}, A_n + 6 \frac{1}{\sqrt{n}}] = 95\%-\text{CI for } \mu\). Yes
Match Items

[1] \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)

[2] \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)

[3] \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)

[4] \( g(\cdot) \text{ convex } \Rightarrow E[g(X)] \geq g(E[X]) \)

[5] \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)

[6] \( \sum_y y \Pr[Y = y | X = x] \)

[7] \( Pr[|\frac{X_1 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0, \)

[8] \( E[(Y - E[Y|X])h(X)] = 0. \)
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)

2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)

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4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)

5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)

6. \( \sum_y y \Pr[Y = y \mid X = x] \)

7. \( \Pr[|\frac{X_1 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \to 0 \)

8. \( E[(Y - E[Y|X])h(X)] = 0 \)

- WLLN
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex } \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y yPr[Y = y|X = x] \)
7. \( Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- WLLN (7)
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0 \)

- WLLN (7)
- MMSE
Match Items

1. $Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)}$
2. $Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2}$
3. $Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$
4. $g(\cdot)$ convex $\Rightarrow E[g(X)] \geq g(E[X])$
5. $E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]).$
6. $\sum_y yPr[Y = y|X = x]$
7. $Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0,$
8. $E[(Y - E[Y|X])h(X)] = 0.$

- WLLN (7)
- MMSE (6)
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex } \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y Pr[Y = y | X = x] \)
7. \( Pr[| \frac{X_1 + \cdots + X_n}{n} - E[X_1] | \geq \varepsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- WLLN (7)
- MMSE (6)
- Projection property
Match Items

[1] \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)

[2] \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)

[3] \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)

[4] \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)

[5] \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)

[6] \( \sum_y y \Pr[Y = y | X = x] \)

[7] \( \Pr[|\frac{X_1 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0 \)

[8] \( E[(Y - E[Y|X])h(X)] = 0 \)

- WLLN (7)
- MMSE (6)
- Projection property (8)
Match Items

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev
Match Items

1. $\Pr[X \geq a] \leq \frac{E[f(X) - f(a)]}{f(a)}$

2. $\Pr[|X - E[X]| > a] \leq \frac{\operatorname{var}[X]}{a^2}$

3. $\Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$

4. $g(\cdot)$ convex $\Rightarrow E[g(X)] \geq g(E[X])$

5. $E[Y] + \frac{\operatorname{cov}(X, Y)}{\operatorname{var}(X)} (X - E[X])$

6. $\sum_y y \Pr[Y = y | X = x]$

7. $\Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon \rightarrow 0,$

8. $E[(Y - E[Y|X])h(X)]] = 0.$

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
Match Items

1. \( Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y Pr[Y = y | X = x] \)
7. \( Pr[\frac{X_1 + \cdots + X_n}{n} - E[X_1] \geq \varepsilon] \rightarrow 0 \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE
Match Items

1. $\Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)}$
2. $\Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2}$
3. $\Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$
4. $g(\cdot)$ convex $\Rightarrow E[g(X)] \geq g(E[X])$
5. $E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)}(X - E[X])$
6. $\sum_y y \Pr[Y = y | X = x]$
7. $\Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0$
8. $E[(Y - E[Y|X])h(X)] = 0$

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \text{ convex} \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]). \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \rightarrow 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov’s inequality
Match Items

1. \( \Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)} \)
2. \( \Pr[|X - E[X]| > a] \leq \frac{\text{var}[X]}{a^2} \)
3. \( \Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}} \)
4. \( g(\cdot) \) convex \( \Rightarrow E[g(X)] \geq g(E[X]) \)
5. \( E[Y] + \frac{\text{cov}(X, Y)}{\text{var}(X)} (X - E[X]) \)
6. \( \sum_y y \Pr[Y = y | X = x] \)
7. \( \Pr[\left| \frac{X_1 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0, \)
8. \( E[(Y - E[Y|X])h(X)] = 0. \)

- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov’s inequality (1)
Quiz 1: G

1. What is \( P[A|B] \)?
   \[ P[A|B] = \frac{P[A \cap B]}{P[B]} = 0.4 \times 0.7 \]

2. What is \( P[B|A] \)?
   \[ P[B|A] = \frac{P[A \cap B]}{P[A]} = 0.4 \]

3. Are \( A \) and \( B \) positively correlated?
   No.
   \[ P[A \cap B] = 0.4 < P[A] \times P[B] = 0.6 \times 0.7 \]
Quiz 1: G

1. What is $P[A | B]$?
   $P[A | B] = \frac{P[A \cap B]}{P[B]} = 0.4 \div 0.7 = 0.57$.

2. What is $P[B | A]$?
   $P[B | A] = \frac{P[A \cap B]}{P[A]} = 0.4 \div 0.2 = 2$.

3. Are $A$ and $B$ positively correlated?
   No. $P[A \cap B] = 0.4 < P[A] \times P[B] = 0.2 \times 0.3$. 

The diagram shows a Venn diagram with probabilities:
- $P[A] = 0.2$
- $P[B] = 0.3$
- $P[A \cap B] = 0.4$
- $P[\Omega - A] = 0.1$
- $P[\Omega - B] = 0.3$
Quiz 1: G

1. What is $P[A|B]$?
Quiz 1: G

1. What is $P[A|B]$?

$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.1}{0.3} = \frac{0.4}{0.7}$.
Quiz 1: G

1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} =$$
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$
1. What is \( P[A|B] \)?

\[
Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}
\]

2. What is \( Pr[B|A] \)?

No. 

\[
Pr[A \cap B] < Pr[A] \cdot Pr[B] = 0.4 \times 0.7
\]
Quiz 1: G

1. What is $P[A|B]$?

   $Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$

2. What is $Pr[B|A]$?

   $Pr[B|A] =$
Quiz 1: G

1. What is $P[A|B]$?

   $Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$

2. What is $Pr[B|A]$?

   $Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.7}$
Quiz 1: G

1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$

3. Are $A$ and $B$ positively correlated?
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$

3. Are $A$ and $B$ positively correlated?

No.
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$

3. Are $A$ and $B$ positively correlated?

No. $Pr[A \cap B] = 0.4 < Pr[A]Pr[B] = 0.6 \times 0.7$. 
Quiz 1: G

4. What is $E[Y|X]$?

$$E[Y|X] = 0 = 1 	imes Pr[Y=0|X=0] + 2 \times Pr[Y=2|X=0]$$

$$= 2 \times 0.4 = 1.0$$

5. What is $cov(X,Y)$?

$$cov(X,Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$$

6. What is $L[Y|X]$?

$$L[Y|X] = E[Y] + cov(X,Y) \times var(X)(X - E[X]) = 1.4 + (-0.04 \times 0.4 \times 0.4) = 1.33$$
4. What is \( E[Y|X] \)?

\[
E[Y|X] = 0 \times \Pr[Y=0|X=0] + 2 \times \Pr[Y=2|X=0] = 2 \times 0.3 = 0.6
\]

5. What is \( \text{cov}(X,Y) \)?

\[
\text{cov}(X,Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04
\]

6. What is \( L[Y|X] \)?

\[
L[Y|X] = E[Y] + \text{cov}(X,Y) \text{var}(X) (X - E[X]) = 1.4 + (-0.04) \times 0.6 \times (X - 0.6)
\]
4. What is $E[Y|X]$?
4. What is $E[Y|X]$?

$$E[Y|X = 0] =$$
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$

= 

\[= \]
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$

$$= 2 \times \frac{0.3}{0.4} = \frac{3}{2}$$
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$

$$= 2 \times \frac{0.3}{0.4} = 1.5$$
4. What is $E[Y|X]$?

\[
E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] = 0 \times 0 + 2 \times \frac{0.3}{0.4} = 1.5
\]

\[
E[Y|X = 1] =
\]
4. What is $E[Y|X]$?

\[
E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] \\
= 2 \times \frac{0.3}{0.4} = 1.5
\]

\[
E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1]
\]
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$
$$= 2 \times \frac{0.3}{0.4} = 1.5$$

$$E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1]$$
$$= 0 \times 0 + 2 \times 0.4$$
$$= 0.8$$

$$= 1.5$$
4. What is $E[Y|X]$?

\[
E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] \\
= 2 \times \frac{0.3}{0.4} = 1.5
\]

\[
E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1] \\
= 2 \times \frac{0.4}{0.6} =
\]
4. What is $E[Y|X]$?

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E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] \\
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\]

\[
E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1] \\
= 2 \times \frac{0.4}{0.6} = 1.33
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E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] \\
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E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1] \\
= 2 \times \frac{0.4}{0.6} = 1.33
\]

5. What is \( \text{cov}(X, Y) \)?
Quiz 1: G

4. What is $E[Y|X]$?

\[
E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0] \\
= 2 \times \frac{0.3}{0.4} = 1.5
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\[
E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1] \\
= 2 \times \frac{0.4}{0.6} = 1.33
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E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]
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= 2 \times \frac{0.3}{0.4} = 1.5
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= 2 \times \frac{0.4}{0.6} = 1.33
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5. What is $cov(X, Y)$?

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cov(X, Y) = E[XY] - E[X]E[Y] =
\]
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$$E[Y|X=0] = 0 \times Pr[Y=0|X=0] + 2 \times Pr[Y=2|X=0]$$

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$$E[Y|X=1] = 0 \times Pr[Y=0|X=1] + 2 \times Pr[Y=2|X=1]$$

$$= 2 \times \frac{0.4}{0.6} = 1.33$$

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$$\text{cov}(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 =$$
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$

$$= 2 \times \frac{0.3}{0.4} = 1.5$$

$$E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1]$$

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5. What is $cov(X, Y)$?

$$cov(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$$
4. What is $E[Y \mid X]$?

\[
E[Y \mid X = 0] = 0 \times Pr[Y = 0 \mid X = 0] + 2 \times Pr[Y = 2 \mid X = 0] = 0 \times 0.3 + 2 \times 0.4 = 1.5
\]

\[
E[Y \mid X = 1] = 0 \times Pr[Y = 0 \mid X = 1] + 2 \times Pr[Y = 2 \mid X = 1] = 0 \times 0.6 + 2 \times 0.4 = 1.33
\]

5. What is $cov(X, Y)$?

\[
cov(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04
\]

6. What is $L[Y \mid X]$?
4. What is $E[Y|X]$?

\[
E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]
\]
\[
= 2 \times \frac{0.3}{0.4} = 1.5
\]

\[
E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1]
\]
\[
= 2 \times \frac{0.4}{0.6} = 1.33
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\text{cov}(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04
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6. What is $L[Y|X]$?
Quiz 1: G

4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$
$$= 2 \times \frac{0.3}{0.4} = 1.5$$

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$$= 2 \times \frac{0.4}{0.6} = 1.33$$

5. What is $cov(X, Y)$?

$$cov(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$$

6. What is $L[Y|X]$?

$$L[Y|X] = E[Y] + \frac{cov(X, Y)}{\text{var}(X)}(X - E[X]) =$$
4. What is $E[Y|X]$?

$$E[Y|X = 0] = 0 \times Pr[Y = 0|X = 0] + 2 \times Pr[Y = 2|X = 0]$$
$$= 2 \times \frac{0.3}{0.4} = 1.5$$

$$E[Y|X = 1] = 0 \times Pr[Y = 0|X = 1] + 2 \times Pr[Y = 2|X = 1]$$
$$= 2 \times \frac{0.4}{0.6} = 1.33$$

5. What is $\text{cov}(X, Y)$?

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$$

6. What is $L[Y|X]$?

$$L[Y|X] = E[Y] + \frac{\text{cov}(X,Y)}{\text{var}(X)} (X - E[X]) = 1.4 + \frac{-0.04}{0.6 \times 0.4} (X - 0.6)$$
Quiz 1: G

7. Is this Markov chains irreducible? 
Yes.

8. Is this Markov chain periodic? 
No.
The return times to 3 are 
\{3, 5, ..\}:
coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? 
Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} m = \frac{1}{11} \{X_m = 3\} \) converge as \( n \to \infty \)? 
Yes!

11. Calculate \( \pi \).
Let \( a = \pi(1) \).
Then 
\[ a = \pi(5), \pi(2) = 0, 5a, \pi(4) = \pi(2) = 0, 5a, \pi(3) = 0, 5\pi(1) + \pi(4) = a. \]
Thus, 
\[ \pi = [a, 0.5a, a, 0.5a] = [1, 0.5, 1, 0.5], \] 
so \( a = \frac{1}{4} \).
Quiz 1: G

7. Is this Markov chains irreducible? 
Yes.

8. Is this Markov chain periodic? 
No.
The return times to 3 are \{3, 5, \ldots\}.

9. Does \(\pi_n\) converge to a value independent of \(\pi_0\)? 
Yes!

10. Does \(\frac{1}{n} \sum_{m=1}^{n-1} m = 1\) \{\(X_m = 3\)\} converge as \(n \to \infty\)? 
Yes!

11. Calculate \(\pi\).
Let \(a = \pi(1)\).
Then \(a = \pi(5)\), \(\pi(2) = 0\), \(5a\), \(\pi(4) = \pi(2) = 0\), \(5a\), \(\pi(3) = 0\), \(5\pi(1) + \pi(4) = a\).
Thus, \(\pi = \begin{bmatrix} a, 0 \\ 5a, a \\ 5a, 1 \end{bmatrix} = \begin{bmatrix} 1, 0 \\ 5, 1 \end{bmatrix}\), so \(a = \frac{1}{4}\).
7. Is this Markov chains irreducible?
Quiz 1: G

7. Is this Markov chains irreducible? Yes.
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No.
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \(\{3, 5, \ldots\}\):
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, ..\}: coprime!
7. Is this Markov chains irreducible? **Yes.**
8. Is this Markov chain periodic?  
   **No.** The return times to 3 are \{3, 5, ..\}: coprime!
9. Does $\pi_n$ converge to a value independent of $\pi_0$?
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
Quiz 1: G

7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?

   No. The return times to 3 are \{3, 5, \ldots\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)?
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic? 
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
9. Does $\pi_n$ converge to a value independent of $\pi_0$? Yes!
10. Does $\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}$ converge as $n \to \infty$? Yes!
11. Calculate $\pi$. 

Let $a = \pi(1)$.
Then $a = \pi(5)$, $\pi(2) = 0.5$, $\pi(3) = 0.5a$, $\pi(4) = \pi(2) = 0.5a$.

Thus, $\pi = \left[a, 0.5a, a, 0.5a\right] = [1, 0.5a, 1, 0.5a]a$, so $a = \frac{1}{4}$. 
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, ..\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).
   Let \( a = \pi(1) \).
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!
11. Calculate \( \pi \).
    Let \( a = \pi(1) \). Then \( a = \pi(5) \),
7. Is this Markov chain irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \( \{3, 5, \ldots\} \): coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!
11. Calculate \( \pi \).
    Let \( a = \pi(1) \). Then \( a = \pi(5), \pi(2) = 0.5a \),
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?

   No. The return times to 3 are \{3, 5, \ldots\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).

   Let \( a = \pi(1) \). Then \( a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \)
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
   
   No. The return times to 3 are \{3, 5, ..\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).

   Let \( a = \pi(1) \). Then \( a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a. \)
7. Is this Markov chain irreducible? Yes.
8. Is this Markov chain periodic?

   No. The return times to 3 are \(\{3, 5, \ldots\}\): coprime!

9. Does \(\pi_n\) converge to a value independent of \(\pi_0\)? Yes!
10. Does \(\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}\) converge as \(n \to \infty\)? Yes!
11. Calculate \(\pi\).

    Let \(a = \pi(1)\). Then \(a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a\). Thus,

    \[
    \pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a, \text{ so } a =
    \]
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?  
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).

   Let \( a = \pi(1) \). Then \( a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a \). Thus, \( \pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a \), so \( a = 1/4 \).
Quiz 1: G

12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5 \]

\[ \beta(2) = 1 + 0.5 \beta(1) \]

\[ \beta(3) = 1 + 0.5 \beta(2) \]

\[ \beta(4) = 1 + 0.5 \beta(3) \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) = 2.5 + 0.5 \beta(1) \]

Hence, \( \beta(1) = 5 \).
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5 \beta(5) \]

\[ \beta(2) = 1 + 0.5 \beta(1) \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + \beta(1)) = 2.5 + 0.5 \beta(1) \]

Hence, \( \beta(1) = 5 \).
12. Write the first step equations for calculating the mean time from 1 to 4.
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5 \beta(2) + 0.5 \beta(3) \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[
\begin{align*}
\beta(1) &= 1 + 0.5\beta(2) + 0.5\beta(3) \\
\beta(2) &= 1 \\
\beta(3) &= 1 + \beta(5)
\end{align*}
\]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[
\beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3)
\]
\[
\beta(2) = 1
\]
\[
\beta(3) = 1 + \beta(5)
\]
\[
\beta(5) = 1 + \beta(1).
\]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[
\begin{align*}
\beta(1) &= 1 + 0.5\beta(2) + 0.5\beta(3) \\
\beta(2) &= 1 \\
\beta(3) &= 1 + \beta(5) \\
\beta(5) &= 1 + \beta(1).
\end{align*}
\]

13. Solve these equations.
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5 \beta(2) + 0.5 \beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
\[ = 2.5 + 0.5\beta(1). \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
\[ = 2.5 + 0.5\beta(1). \]

Hence, \( \beta(1) = 5. \)
14. Which is $E[Y|X]$? Blue, red or green?
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. 
Quiz 1: G

14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.

Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x) + b(x)}{2}$. 
15. Which is $L[Y|X]$? Blue, red or green?

Answer: Blue. Cannot be red (not a straight line). Cannot be green: $X$ and $Y$ are clearly positively correlated.
15. Which is $L[Y|X]$? Blue, red or green?

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Cannot be red (not a straight line).
Quiz 1: G

15. Which is $L[Y|X]$? Blue, red or green?

Answer: Blue. Cannot be red (not a straight line). Cannot be green: $X$ and $Y$ are clearly positively correlated.
Quiz 2: PG

1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need
\[
\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]
\]
That is,
\[
0.2 = (y + 0.3) \times 0.5
\]
Hence,
\[
y = 0.2\] and \[x = 0.3\].

2. Find the value of \(x\) that maximizes \(\Pr[B|A]\).

Obviously, it is \[x = 0.5\].
Quiz 2: PG

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   \]

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   y = 0.2 \text{ and } x = 0.3.
   \]

2. Find the value of \(x\) that maximizes \(Pr[B|A]\).

   Obviously, it is \(x = 0.5\).
Quiz 2: PG

3. Find $\alpha$ so that $X$ and $Y$ are independent. We need $\Pr[X=0, Y=0] = \Pr[X=0] \cdot \Pr[Y=0]$. That is, $0.1 = (0.1 + \alpha) \cdot (0.1 + 0.2) = 0.03 + 0.3 \alpha$. Hence, $\alpha = 0.233$. 
Quiz 2: PG

Find $\alpha$ so that $X$ and $Y$ are independent.

We need $\Pr[X = 0, Y = 0] = \Pr[X = 0] \Pr[Y = 0]$

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Hence,

$$\alpha = 0.233$$
Quiz 2: PG

4. A CS70 student is great with probability 0.3 and good with probability 0.7. A great student solves each question correctly with probability 0.8 whereas a good student does it with probability 0.6. One student got right 70% of the 10 questions on Midterm 1 and 70% of the 10 questions on Midterm 2. What is the expected score of the student on the final?

\[ p = \text{Pr}\left[ \text{great} \mid \text{scores} \right] = 0.3 (0.8) + 0.7 (0.6) = 0.6 + 0.4 = 0.27 \]

Expected score = 80% + (1 - p) 60% ≈ 65%.
4. A CS70 student is great w.p. 0.3 and good w.p. 0.7.
Quiz 2: PG

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Quiz 2: PG

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Expected score = \(p80\% + (1 - p)60\% \approx \)
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\[ = \frac{(0.3) 0.8^{14} 0.2^6}{(0.3) 0.8^{14} 0.2^6 + (0.7) 0.6^{14} 0.4^6} \approx 0.27 \]

Expected score \[ = p80\% + (1 - p)60\% \approx 65\%. \]
You roll a balanced six-sided die 20 times. Use CLT to upper-bound the probability that the total number of dots exceeds 85.

Let $X = X_1 + \cdots + X_{20}$ be the total number of dots. Then

$$X - 70 \approx N(0, 1)$$

where $\sigma^2 = \text{var}(X_1) = \frac{1}{6} \sum_{m=1}^{6} m^2 - (3.5)^2 \approx 2.9 = 1.7^2$.

Now,

$$\Pr[X > 85] = \Pr[X - 70 > 151.7 \times 4.5] = \Pr[X - 70 > 2 \times 1.7] \approx 2.5\%.$$
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Let $X = X_1 + \cdots + X_{20}$ be the total number of dots.
Quiz 2: PG

5. You roll a balanced six-sided die 20 times. Use CLT to upper-bound the probability that the total number of dots exceeds 85.

Let \( X = X_1 + \cdots + X_{20} \) be the total number of dots. Then

\[
\frac{X - 70}{\sigma \sqrt{20}} \approx \mathcal{N}(0, 1)
\]

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\]

Now,
\[
\Pr[X > 85] = \Pr[X - 70 > 15] = \Pr\left[ \frac{X - 70}{1.7 \times 4.5} > \frac{15}{1.7 \times 4.5} \right]
\]
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Now,

$$Pr[X > 85] = Pr[X - 70 > 15]$$

$$= Pr[\frac{X - 70}{1.7 \times 4.5} > \frac{15}{1.7 \times 4.5}]$$

$$= Pr[\frac{X - 70}{1.7 \times 4.5} > 2]$$

$$\approx 2.5\%.$$
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$$\Pr[X > 85] = \Pr[X - 70 > 15]$$

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$$= \Pr\left[\frac{X - 70}{1.7 \times 4.5} > 2\right] \approx 2.5\%.$$
You roll a balanced six-sided die 20 times. Use Chebyshev to upper-bound the probability that the total number of dots exceeds 85. Let \( X = X_1 + \cdots + X_{20} \) be the total number of dots. Then \( \Pr[X > 85] = \Pr[|X - 70| > 15] \leq \frac{\text{var}(X)}{15^2} \). Now, \( \text{var}(X) = 20 \times \text{var}(X_1) = 20 \times 2.9 = 58 \). Hence, \( \Pr[X > 85] \leq \frac{58}{15^2} \approx 0.26 \).
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Let \( X = X_1 + \cdots + X_{20} \) be the total number of dots. Then

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Pr[X > 85] = Pr[X - 70 > 15] \leq Pr[|X - 70| > 15]
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Let $X = X_1 + \cdots + X_{20}$ be the total number of dots. Then

$$Pr[X > 85] = Pr[X - 70 > 15] \leq Pr[|X - 70| > 15] \leq \frac{\text{var}(X)}{15^2}.$$
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Now,

$$var(X) = 20var(X_1) = 20 \times 2.9 = 58.$$ 

Hence,

$$Pr[X > 85] \leq \frac{58}{15^2} \approx 0.26.$$
7. Let \( X, Y, Z \) be i.i.d. Expo \((1)\).

Find \( L[X | X + 2Y + 3Z] \).

Let \( V = X + 2Y + 3Z \).

One finds \( L[X | V] = E[X] + \text{cov}(X, V) \text{var}(V) (V - E[V]) \).

\( E[X] = 1, E[V] = 6, \text{cov}(X, V) = \text{var}(X) = 1 \).

\( \text{var}(V) = 1 + 4 + 9 = 14 \).

Hence, \( L[X | V] = 1 + \frac{1}{14} (V - 6) \).

8. Let \( X, Y, Z \) be i.i.d. Expo \((1)\). Calculate \( E[X + Z | X + Y] \).

\( E[X + Z | X + Y] = E[X | X + Y] + E[Z] = \frac{1}{2}(X + Y) + 1 \).

9. Let \( X, Y, Z \) be i.i.d. Expo \((1)\). Calculate \( L[X + Z | X + Y] \).

\( L[X + Z | X + Y] = \frac{1}{2}(X + Y) + 1 \).
Quiz 2: PG

7. Let $X, Y, Z$ be i.i.d. $Expo(1)$.
Let $X, Y, Z$ be i.i.d. $\text{Expo}(1)$. Find $L[X|X + 2Y + 3Z]$. 

$V = X + 2Y + 3Z$. One finds $L[X|V] = E[X] + \text{cov}(X, V) \var(V) (V - E[V])$. 

$E[X] = 1$, $E[V] = 6$, $\text{cov}(X, V) = \text{var}(X) = 1$, $\var(V) = 14$. 

Hence, $L[X|V] = 1 + \frac{1}{14} (V - 6)$. 

Let $X, Y, Z$ be i.i.d. $\text{Expo}(1)$. Calculate $E[X + Z|X + Y]$. 

$E[X + Z|X + Y] = E[X|X + Y] + E[Z] = \frac{1}{2} (X + Y) + 1$. 

Let $X, Y, Z$ be i.i.d. $\text{Expo}(1)$. Calculate $L[X + Z|X + Y]$. 

$L[X + Z|X + Y] = \frac{1}{2} (X + Y) + 1$. 

Quiz 2: PG
Quiz 2: PG

7. Let $X, Y, Z$ be i.i.d. $Expo(1)$. Find $L[X|X + 2Y + 3Z]$.
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Quiz 2: PG

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Let $V = X + 2Y + 3Z$. One finds

$$E[X|V] = E[X|X + 2Y + 3Z] = \frac{1}{2}(X + Y) + 1.$$
7. Let $X, Y, Z$ be i.i.d. $\text{Expo}(1)$. Find $L[X \mid X + 2Y + 3Z]$. Let $V = X + 2Y + 3Z$. One finds

$$L[X \mid V] =$$
Quiz 2: PG


Let $V = X + 2Y + 3Z$. One finds

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10. You roll a balanced die.
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Every time you get a 6, your fortune is multiplied by 10.
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Every time you get a 6, your fortune is multiplied by 10.
Every time you do not get a 6, your fortune is divided by 2.
Let $X_n$ be your fortune at the start of step $n$, 

Calculate $E[X_n]$. 

We have $X_1 = 1.$

Also, $E[X_{n+1} | X_n] = X_n \times \left(\frac{1}{6} \times 10 + \frac{5}{6} \times \frac{1}{2}\right) = \rho X_n$, 

$\rho = \frac{1}{6} \times 10 + \frac{5}{6} \times \frac{1}{2} \approx 2.1$.

Hence, $E[X_{n+1}] = \rho E[X_n]$,

Thus, $E[X_n] = \rho^{n-1} E[X_1]$, $n \geq 1$. 

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Thus,

$$E[X_n] = \rho^{n-1}, n \geq 1.$$
Quiz 3: R

1. The lifespans of good lightbulbs are exponentially distributed with mean 1 year. Those of defective bulbs are exponentially distributed with mean 0.8.

All the bulbs in one batch are equally likely to be good or defective.

You test one bulb and note that it burns out after 0.6 year.

(a) What is the probability you got a batch of good bulbs?

(b) What is the expected lifespan of another bulb in that batch?

Hint: If \( X = \text{Expo}(\lambda) \), then
\[
    f_X(x) = \lambda e^{-\lambda x} \mathbb{1}_{\{x > 0\}},
\]
\[
    E[X] = \frac{1}{\lambda}.
\]

Let \( X \) be the lifespan of a bulb, \( G \) the event that it is good, and \( B \) the event that it is bad.

(a) \[
    p := \Pr[X \in (0.6, 0.6 + \delta) | G] = 0.5
\]

\[
    \Pr[X \in (0.6, 0.6 + \delta) | G] = 0.5 \Pr[X \in (0.6, 0.6 + \delta)] + 0.5 \Pr[X \in (0.6, 0.6 + \delta) | D] = e^{-0.6 \delta} e^{-0.6 \delta} + (0.8)^{-1} e^{-(0.8) - 0.6 \delta} \approx 0.488.
\]

(b) \[
    E[\text{lifespan of other bulb}] = p \times 1 + (1 - p) \times 0.8 \approx 0.9.
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Let $X$ be the lifespan of a bulb, $G$ the event that it is good, and $B$ the event that it is bad.

(a) $p := \Pr[G|X \in (0.6, 0.6+\delta)] = 0.5$

$\Pr[X \in (0.6, 0.6+\delta)|G] + 0.5 \Pr[X \in (0.6, 0.6+\delta)|D] = 0.488$.

(b) $E[\text{lifespan of other bulb}] = p \times 1 + (1-p) \times 0.8 \approx 0.9$. 
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**Hint:** If $X = Expo(\lambda), f_X(x) = \lambda e^{-\lambda x}1\{x > 0\}, E[X] = 1/\lambda.$
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(b) $E[\text{lifespan of other bulb}] = p \times 1 + (1-p) \times 0.8 \approx 0.9$. 
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Quiz 3: R

2. In the figure, $1, 2, 3, 4$ are links that fail after i.i.d. times that are $U[0,1]$. Find the average time until $A$ and $B$ are disconnected.

Let $X_k$ be the lifespan of link $k$, for $k = 1, \ldots, 4$. We are looking for $E[Z]$ where $Z = \max\{Y_1, Y_2\}$ with $Y_1 = \min\{X_1, X_2\}$ and $Y_2 = \min\{X_3, X_4\}$.

$$Pr[Y_1 > t] = Pr[X_1 > t] Pr[X_2 > t] = (1 - t)^2$$

$$Pr[Z \leq t] = Pr[Y_1 \leq t] Pr[Y_2 \leq t] = (1 - (1 - t)^2)^2 = (2t - t^2)^2$$

$$f_Z(t) = 8t - 12t^2 + 4t^3$$

$$E[Z] = \int_0^1 tf_Z(t) \, dt = \frac{81}{3} - \frac{121}{4} + \frac{41}{5} \approx 0.4667.$$
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E[Z] = \int_0^1 tf_Z(t) \, dt = \frac{81}{3} - \frac{121}{4} + \frac{41}{5} = 0.4667.
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2. In the figure, 1, 2, 3, 4 are links that fail after i.i.d. times that are $U[0, 1]$.

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Quiz 3: R

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We are given $\pi_0$.

Find $\lim_{n \to \infty} \pi_n$.

With probability $\alpha := 0.2 \pi_0 (1) + \pi_0 (2) + \pi_0 (3)$, the MC ends up in $\{2, 3\}$.

With probability $1 - \alpha$, it ends up in state 4.

If it is in $\{2, 3\}$, the probability that it is in state 2 converges to $0.20 + 0.6 = 0.25$.

Hence, the limiting distribution is $[0, 0.25 \alpha, 0, 0.75 \alpha, 1 - \alpha]$. 
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4. A bag has $n$ red and $n$ blue balls. You pick two balls (no replacement). Let $X = 1$ if ball 1 is red and $X = -1$ otherwise. Define $Y$ likewise for ball 2.

→ Are $X$ and $Y$ positively, negatively, or uncorrelated?

Clearly, negatively.

5. Calculate $\text{cov}(X, Y)$.

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y],$$

by symmetry $E[X] = 0$.

$$E[XY] = \Pr[X = Y] - \Pr[X \neq Y] = \frac{n-1}{2n-1}$$

E.g., if $X = +1 = \text{red}$, then $Y$ is red w.p. $\frac{n-1}{2n-1}$.

$$E[XY] = \frac{2}{2n-1} - 1 = -1,$$

Thus, $\text{cov}(X, Y)$. 

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6. What is \( L[Y|X] \)? \( L[Y|X] = -\frac{1}{2n-1} X \). Indeed, \( \text{var}(X) = 1 \),
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\( \rightarrow \) Are \( X \) and \( Y \) positively, negatively, or uncorrelated?

Clearly, negatively.

5. Calculate \( \text{cov}(X, Y) \).

\[
\text{cov}(X, Y) = E[XY] - E[X]E[Y]
\]

\[
E[X] = E[Y], \text{ by symmetry}
\]

\[
E[X] = 0
\]

\[
E[XY] = Pr[X = Y] - Pr[X \neq Y] = 2Pr[X = Y] - 1
\]

\[
Pr[X = Y] = (n-1)/(2n-1)
\]

E.g., if \( X = +1 = \text{red} \), then \( Y \) is red w.p. \( (n-1)/(2n-1) \)

\[
E[XY] = 2(n-1)/(2n-1) - 1 = -1/(2n-1) = \text{cov}(X, Y).
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6. What is \( L[Y|X] \)? \( L[Y|X] = -\frac{1}{2n-1} X \). Indeed, \( \text{var}(X) = 1 \), obviously!
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Since \( X \) takes only two values, any \( g(X) \) is linear in \( X \). Hence, \( E[Y | X] = L[Y | X] \).

Alternatively, let \( \alpha = \Pr[X = Y] = \frac{(n-1)(2n-1)}{2n^2} \).

Then,
\[
E[Y | X = 1] = \alpha - (1 - \alpha) = 2\alpha - 1,
\]
\[
E[Y | X = -1] = -\alpha + (1 - \alpha) = 1 - 2\alpha.
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Thus,
\[
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Hence, $E[Y|X] = \text{L}[Y|X]$. 

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$E[Y|X=1] = \alpha - (1-\alpha) = 2\alpha - 1$, 

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