CS 70 Discrete Mathematics for CS
Spring 2006 Vazirani

HW 7

Due Friday, March 17

For homework this week, please re-do problems 1 (first 3 parts), 2 and 7 from the midterm. Even if you got full credit on these questions on your midterm, please try to write your solutions as succintly as possible. All of these problems can be done in 3 or 4 lines. If you got full credit and feel like your solution is already very succint, feel free to hand in a photocopy of your solution. In addition, this homework contains one question about Eulerian paths and tours.

For reference, three midterm problems are included here again.

1. **Quantifiers**

   For each of the following, indicate whether the two statements are equivalent.

   (a) $\forall x\ (P(x) \vee Q(x))$ and $(\forall x\ P(x)) \vee (\forall x\ Q(x))$.

   (b) $\forall x\ (P(x) \wedge Q(x))$ and $(\forall x\ P(x)) \wedge (\forall x\ Q(x))$.

   (c) $\forall x\ \neg P(x)$ and $\neg \forall x\ P(x)$.

2. **Induction**

   Suppose you know that $P(1)$ is true, and that $\forall k \geq 1, P(k) \implies P(2k)$. Use induction to show that $P(n)$ is true whenever n is a power of 2.

3. **Chinese Remainder Theorem**

   Calculate $31^{4,801} \bmod 35$, using the Chinese remainder theorem. Show your work.

   Hint: $3^6 \equiv 1 \bmod 7$.

4. **Eulerian Paths and Cycles**

   We proved in class that a connected (undirected) graph has an Eulerian cycle if and only if every vertex has even degree.

   (a) Prove that if a graph G on n vertices has exactly $2d$ vertices of odd degree, then there are d walks (i.e. paths that can go through the same vertex more than once) that *together* cover all the edges of G (i.e., each edge of G occurs in exactly one of the d walks; and each of the walks should not contain any particular edge more than once).

   (b) Suppose we have an (undirected) graph in which all vertices have even degree. Briefly describe how you would give a direction to each edge such that the indegree equals the outdegree of every vertex in the resulting directed graph.