CS 70 Discrete Mathematics for CS
Spring 2006 Vazirani

HW 9

Due Tuesday, April 4

1. **Monty Hall**

   (a) After many years, the standard version of the game with three doors becomes a little boring, so Monty decides to increase the number of doors to four (with one prize and thr ee goats). What is now the probability that the contestant wins under the switching strategy?

   (b) Finally, suppose there are n doors and $m \leq n-2$ prizes (each one behin d a different color). Again, the contestant picks a door and Monty then opens a goat door. (The condition $m \leq n-2$ ensures that this is always possible.) What is the probability of winning under the "sticking" strategy? What is the probability of winning under the "switching" strategy? Both of your answers should be given as a function of m and n.

2. **Independence**
   Let Ω be a sample space, and let $A, B \subseteq \Omega$ be two *independent* events. Let $\overline{A} = \Omega - A$ and $\overline{B} = \Omega - B$ (sometimes written $\neg A$ and $\neg B$) denote the complementary events.

   For the purposes of this question, you may use the following definition of independence: Two events A, B are *independent* if $\Pr[A \cap B] = \Pr[A]\Pr[B]$.

   (a) Prove or disprove: $\overline{A}$ and $\overline{B}$ are necessarily independent.

   (b) Prove or disprove: A and $\overline{B}$ are necessarily independent.

   (c) Prove or disprove: A and $\overline{A}$ are necessarily independent.

   (d) Prove or disprove: It is possible that $A = B$.

3. **(10 pts.) Tongue twisters**
   Recall that an *anagram* of a word is a string made up from the letters of that word, in any order. (For instance, there are exactly three anagrams of EYE: namely, EEY, EYE, and YEE. Note that anagrams need not form legal words in any language.)

   How many different anagrams of SEASHELLS are there? Justify your answer.

4. **Marbles**
   Box A contains 1 black and 3 white marbles, and box B contains 2 black and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

   - What is the probability that the marble is black?
   - Given that the marble is white, what is the probability that it came from the box A?

5. **Money bags**
   I have a bag containing either a \$1 or \$5 bill (with equal probability assigned to both possibilities). I then add a \$1 bill to the bag, so it now contains two bills. The bag is shaken, and you draw out a \$1

bill. If a second student draws the remaining bill from the bag, what is the chance that it is a $1 bill? Show your work.

6. (20 pts.) Dice (Extra-Credit)

Show how to label the sides of three six-sided dice A, B, and C so that

(a) when you roll dice A and B, the number that comes up on die A is on average larger than the number that comes up on die B

(b) when you roll dice B and C, the number that comes up on die B is on average larger than the number that comes up on die C

(c) when you roll dice C and A, the number that comes up on die C is on average larger than the number that comes up on die A.

The labels on the sides of any one die do not have to be distinct. Compute the probability that the number that comes up on die A is larger than the number that comes up on die B for the dice you construct, and repeat this calculation for the pair B and C and the pair C and A (by the definition of "on average," each of these three probabilities should be at least $1/2$.)