Problem Set 1

1. **Exclusive OR**
   The exclusive OR (written as XOR or $\oplus$) is just what it sounds like: $P \oplus Q$ is true when exactly one of P, Q is true.

   Show, using a truth table, that $P \oplus Q$ is equivalent to $(P \vee Q) \wedge \neg(P \wedge Q)$.

2. **Everyday logic**
   Let's try to apply the language of Logic and Propositions that we learned in class to the rag "Everybody loves my baby, but my baby don't love nobody but me" (by Jack Palmer, ca. the 1920s). Suppose that our universe has one relationship, LOVES, and two distinguished individuals, ME and MYBABY. For example, LOVES(ME, MYBABY) means "I love my baby" and $\exists x \neg$ LOVES(x, ME) means "not everybody loves me."

   - Write a proposition, using this vocabulary and the Boolean connectives $\wedge$, $\vee$, $\neg$, $\Rightarrow$, as well as quantifiers $\forall$ and $\exists$, to transcribe this verse into the language of Logic.
   - Look now at what you wrote, and argue that this proposition implies ME = MYBABY !

3. Prove or disprove:

   (a) if x is irrational, then px is irrational ($\forall p \in \mathbb{Q}$).

   (b) $5n^2 > n!$ ($\forall n \in \mathbb{N}$).

   (c) $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$ ($\forall n \in \mathbb{N}, n \geq 2$).

   (d) $\prod_{i=2}^{n}(1 - \frac{1}{i}) = \frac{1}{n}$ ($\forall n \in \mathbb{N}$). (**hint.** Use mathematical induction)

4. Use mathematical induction to show that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.