Discussion 2A

1. Write the following statements using the notation covered in class. Use $\mathbb{N}$ to denote the set of natural numbers and $\mathbb{Z}$ to denote the set of integers. Also write $P(n)$ for the statement "n is odd".

   (a) For all natural numbers n, $2n$ is even.

   (b) For all natural numbers n, n is odd if n^2 is odd.

   (c) There are no integer solutions to the equation $x^2 - y^2 = 10$.

2. Which of the following statements are true? Let $Q(n)$ be the statement "n is divisible by 2." $\mathbb{N}$ denotes the set of natural numbers.

   (a) $\exists\, k \in \mathbb{N},\ Q(k) \wedge Q(k+1)$.

   (b) $\forall\, k \in \mathbb{N},\ Q(k) \implies Q(k^2)$.

   (c) $\exists\, x \in \mathbb{N}, \neg(\exists\, y \in \mathbb{N}, y < x)$.

3. Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

4. You are on an island inhabited by two types of people: the Liars and the Truthtellers. Liars always make false statements, and Truthtellers always make true statements. In all other respects, the two types are indistinguishable. You meet an attractive local and ask him/her on a date. The local responds, "I will go on a date with you if and only if I am a Truthteller." What does this mean for you?

5. Prove that if you put $n+1$ apples into n boxes, any way you like, then at least one box must contain at least 2 apples. This is known as the *pigeonhole principle*.

6. Prove that the length of the hypotenuse of a non-degenerate right triangle is strictly less than the sum of the two remaining sides.

   (a) Write down the definition of a right triangle and the claim to be proven in mathematical notation.

   (b) Prove the statement by contradiction.

   (c) Prove the statement directly.