EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai **Discussion 2B**

Induction

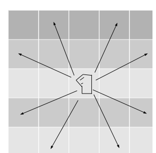
1. **Fun with Binary.** Prove the following statement:

$$\forall n \in \mathbb{N}, \sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

2. **Divergence of harmonic sum.** You may have seen the series $1 + \frac{1}{2} + \frac{1}{3} + \ldots$ in calculus. This is known as a *harmonic series*, and it diverges, i.e. the sum approaches infinity. We are going to prove this fact using induction.

   Let $H_j = \sum_{k=1}^{j} \frac{1}{k}$. Use mathematical induction to show that, for all integers $n \geq 0$, $H_{2^n} \geq 1 + \frac{n}{2}$, thus showing that H_j must grow unboundedly as $j \to \infty$.

3. **Reachability of knight figure in chess.** A knight in chess can move in L-shapes: that is, either 1 space horizontally (either left or right) and 2 spaces vertically (either up or down) or 1 space vertically and 2 spaces horizontally:



   Suppose we have an infinite chessboard, with squares labeled according to their position as (m,n) where m and n are nonnegative integers. A sample of the lower-left corner of the board with labels is below:

(0,2)	(1,2)	(2,2)
(0,1)	(1,1)	(2,1)
(0,0)	(1,0)	(2,0)

   Use induction to show that a knight starting at $(0,0)$ can visit every square (Hint: induct on $r = m+n$).

4. **More squares.** Prove the following statement:

$$\forall n \in \mathbb{N}, \exists m \in \mathbb{N}, \sum_{k=0}^{n} k^3 = m^2$$