EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai Discussion 9A

1. **Rolling dice, revisited**

Let's say you have a fair die with 6 faces numbered 1 through 6. Because the die is fair, you know that each time you roll the die, you are equally likely to get each of the 6 possible outcomes 1 through 6. Now let's say you roll the die 108 million times, constituting 108 million *trials*, as the experts would call it.

In how many of these trials would you expect to see:

(a) The outcome 5 (that is, the number 5 pops up when you roll the die)

(b) An even number outcome

(c) An odd number outcome

(d) A perfect square outcome

Now let's say you have two fair dice instead. And you roll both of them together. In this case, your outcome will be a pair of integers, with the first number in the pair being the reading from the first die and the second number being the reading from the second die. There are thus 36 possible outcomes, $(1,1)$ through $(6,6)$, all equally likely. Imagine you now conduct 108 million trials again, with each trial giving you a pair of integers as the outcome. Let's define the sum of such an outcome to be the sum of the two integers in the pair constituting the outcome.

Now, in how many of these trials would you expect to see:

(e) An outcome with the sum 5

(f) An outcome with even sum

(g) An outcome with odd sum

(h) An outcome with a sum that is a perfect square

(i) An outcome where the two numbers in the pair are relatively prime

2. **Clinical tests**

You may want to use a calculator for this problem.

Let's say that there is a rare disease, and only $p = 1\%$ of the human population has that disease.

Let's also assume that there is a clinical test available for the disease, but the test is not perfect. The chances that the test is accurate are only $q = 90\%$. So, if 1000 people who have the disease are tested, the test will come out positive in only about 900 of these cases. Likewise, if 1000 healthy individuals are tested, the test will come back negative in only about 900 of these cases.

(a) Suppose the above test is done on a population of 10000 individuals. How many individuals do you expect will:

i. Have the disease and test positive for it?
ii. Have the disease, but test negative for it?
iii. Not have the disease, but test positive for it?
iv. Not have the disease, and test negative for it?

(b) Suppose an individual drawn randomly from this population tests positive. What are the chances that this individual actually has the disease?

(c) Suppose an individual drawn randomly from this population tests negative. What are the chances that this individual does not actually have the disease?

(d) Suppose a group of scientists get together and develop a new test that has an accuracy of $q = 99.9\%$ (a major improvement that replaces the old test overnight). Now how do the chances in parts (b) and (c) above change?