1. Sanity Check!

   a. Prove or give a counterexample: for any random variables $X$ and $Y$, $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$.

   b. Derive Chebyshev’s inequality using Markov’s inequality for some random variable $X$.

2. Balls and Bins Again

   For this problem we toss $m$ balls into $n$ bins.

   a. What is the expected number of collisions?

   b. Now, let’s define $X$ to be the number of collisions. At what threshold of collisions $c$ can we ensure that the probability of having more than $c$ collisions is less than $1/2n$?
3. Coin flips

a. Suppose we flip a fair coin $n$ times and we wish to understand the probability that we get at least $3n/4$ heads. Use Markov’s inequality to come up with an upper bound for this probability.

b. Use Markov’s inequality to come up with a similar upper bound on the probability that the number of heads is at least $n$.

c. Find the true probability that there are at least $n$ heads in a sequence of $n$ fair coin flips. Is the bound you derived in the previous part tight?

4. More coin flips

a. Suppose we flip a biased coin 100 times and $X$ is the number of heads we get. We know that $\text{Var}[X] = 16$. What are the possible values for the expected value of $X$?

b. Now suppose $E[X] = 20$. Use Chebyshev’s inequality to derive an upper bound on $\Pr[X \geq 40]$. 