1. Truth tables

Use truth tables to show the following identities (note that the first two are known as De Morgan’s Laws):

1. \( \neg(A \lor B) \equiv \neg A \land \neg B. \)
2. \( \neg(A \land B) \equiv \neg A \lor \neg B. \)
3. \( A \iff B \equiv (A \land B) \lor (\neg A \land \neg B). \)
4. \( (A \Rightarrow (B \Rightarrow C)) \lor (B \Rightarrow (A \land C)) \equiv \neg A \lor \neg B \lor C. \)

2. Writing in propositional logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

1. The square of a nonzero integer is positive.
2. There are no integer solutions to the equation \( x^2 - y^2 = 10. \)
3. There is one and only one real solution to the equation \( x^3 + x + 1 = 0. \)
4. For any two distinct real numbers, we can find a rational number in between them.
3. Implication

Which of the following implications are true? Give a counterexample for each false assertion.

1. \( \forall x \forall y P(x,y) \) implies \( \forall y \forall x P(x,y) \).
2. \( \exists x \exists y P(x,y) \) implies \( \exists y \exists x P(x,y) \).
3. \( \forall x \exists y P(x,y) \) implies \( \exists y \forall x P(x,y) \).
4. \( \exists x \forall y P(x,y) \) implies \( \forall y \exists x P(x,y) \).

4. Proof by contraposition

Let \( x \) be a positive real number. Prove that if \( x \) is irrational (i.e., not a rational number), then \( \sqrt{x} \) is also irrational.

5. Proof by cases

A perfect square is an integer \( n \) of the form \( n = m^2 \) for some integer \( m \). Prove that every odd perfect square is of the form \( 8k + 1 \) for some integer \( k \).