1. **Triangle Inequality**

Recall the triangle inequality, which states that for real numbers $x_1$ and $x_2$,

$$|x_1 + x_2| \leq |x_1| + |x_2|.$$

Use induction to prove the generalized triangle inequality:

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|.$$

2. **Power Inequality**

Use induction to prove that for all integers $n \geq 1$, $2^n + 3^n \leq 5^n$.

3. **Convergence of Series**

Use induction to prove that for all integers $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{3k^{3/2}} \leq 2.$$

*Hint:* Strengthen the induction hypothesis to

$$\sum_{k=1}^{n} \frac{1}{3k^{3/2}} \leq 2 - \frac{1}{\sqrt{n}}.$$
4. Grid Induction

A bug lives on the grid \( \mathbb{N}^2 \). He starts at some location \((i, j) \in \mathbb{N}^2\), and every second he does one of the following (if possible):

(i) Jump one inch down to \((i, j - 1)\), as long as \((i, j - 1) \in \mathbb{N}^2\).
(ii) Jump one inch left to \((i - 1, j)\), as long as \((i - 1, j) \in \mathbb{N}^2\).

For example, if the bug is at \((5, 0)\), then his only option is to jump left to \((4, 0)\). Prove that no matter where the bug starts and how the bug jumps, he will always reach \((0, 0)\) in finite time.

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Supplemental Problems

5. Divergence of Harmonic Series

You may have seen the harmonic series \( 1 + \frac{1}{2} + \frac{1}{3} + \cdots \) in calculus. We will prove that the harmonic series diverges, i.e., the sum approaches infinity.

Let \( H_j = \sum_{k=1}^{j} \frac{1}{k} \). Use induction to show that for all integers \( n \geq 0 \), \( H_{2^n} \geq 1 + \frac{n}{2} \), thus showing that \( H_j \) must grow unboundedly as \( j \to \infty \).

6. Fibonacci Expansion

The Fibonacci numbers are defined recursively by \( F_1 = F_2 = 1 \), and \( F_k = F_{k-1} + F_{k-2} \) for \( k \geq 3 \).

Prove that every positive integer \( n \) has a binary expansion in the Fibonacci basis that does not use two consecutive Fibonacci numbers, i.e., we can write:

\[
n = c_k \cdot F_k + c_{k-1} \cdot F_{k-1} + \cdots + c_2 \cdot F_2 + c_1 \cdot F_1
\]

for some \( k \in \mathbb{N} \) and \( c_1, \ldots, c_k \in \{0, 1\} \) with the property that \( c_i \cdot c_{i+1} = 0 \) for all \( 1 \leq i \leq k - 1 \).

For example, we could write \( 6 = F_1 + F_3 + F_4 \), but this uses consecutive Fibonacci numbers. We can write it instead as \( 6 = F_1 + F_5 \), which satisfies the desired property.