1. **Sanity Check!**

   1. What’s the minimum number of roots of a degree 4 real polynomial?

   2. What’s the minimum number of roots of a degree 5 real polynomial?

   3. What’s the minimum number of roots of a degree 5 polynomial in GF(7)?

   4. How many degree at most 3 polynomials are there in GF(7)?

2. **Lagrange Interpolation**

   Find a unique real polynomial $p(x)$ of degree at most 3 that passes through points $(-1, 3), (0, 1), (1, 2)$, and $(2, 0)$ using Lagrange interpolation.

   1. Find $\Delta_{-1}(x)$ where $\Delta_{-1}(0) = \Delta_{-1}(1) = \Delta_{-1}(2) = 0$ and $\Delta_{-1}(-1) = 1$.

   2. Find $\Delta_0(x)$ where $\Delta_0(-1) = \Delta_0(1) = \Delta_0(2) = 0$ and $\Delta_0(0) = 1$.

   3. Find $\Delta_1(x)$ where $\Delta_1(-1) = \Delta_1(0) = \Delta_1(2) = 0$ and $\Delta_1(1) = 1$.

   4. Find $\Delta_2(x)$ where $\Delta_2(-1) = \Delta_2(0) = \Delta_2(1) = 0$ and $\Delta_2(2) = 1$. 
5. Construct $p(x)$ using a linear combination of $\Delta_{-1}(x), \Delta_0(x), \Delta_1(x)$ and $\Delta_2(x)$.

3. Secret Sharing

Umesh wants to share a secret among 4 TAs and 14 readers, such that a subset of them can reconstruct the secret iff it contains either (i) at least 2 TAs, or (ii) at least 1 TA and at least 2 readers, or (iii) at least 4 readers. Explain how this can be accomplished.

4. Polynomial Intersections

Find (and prove) an upper-bound on the number of times two distinct degree $d$ polynomials can intersect. What if the polynomials’ degrees differ?