1. Sanity check!

At UC Berkeley, each student has an 8-digit ID number.

1. How many ID cards can be formed if repetition of digits is allowed?

2. How many ID cards can be formed if repetition of digits is not allowed?

2. West Dakotan license plates

In the fictional state of West Dakota, license plates consist of 5 characters, each of which must be either a letter (one of the 26 from A through Z) or a digit (one of the 10 from 0 through 9). Beyond this, there are no further restrictions. How many possible West Dakotan license plates…

a. contain only letters?

b. have exactly three letters and two numbers?

c. contain the string ABC?

d. have at least two of the same character?

3. Poker hands

Let's play a version of poker where each hand consists of 6 cards. As usual, the order of cards in each hand doesn't matter. Count the number of hands that have:

a. 3 distinct pairs. E.g. (1H, 1D, 2C, 2S, 3D, 3S) but not (1H, 1D, 1C, 1S, 2D, 2S)

b. Exactly 1 triple and 1 pair. E.g. (1H, 1D, 1C, 2S, 2D, 4S) but not (1H, 1D, 1C, 2S, 2D, 2C) or (1H, 1D, 1C, 1S, 2D, 2C).

First choose 3 distinct numbers (out of 13) without order, and then count the number of ways to designate one to be the triple, one to be the pair and one to be the single. Then pick the suits for each of them.

c. The same question as above, but with a different method:

First choose 3 distinct numbers *with* order. Let these numbers be a, b and c. Now let the deck of cards be of the form $a\_, a\_, a\_, b\_, b\_, c\_$, where the blanks represent suits. Now count the number of ways to choose the suits.

d. Do these two methods give the same answer?

4. Stars and bars

a. Suppose you have seven stars as below:

$$\star\star\star\star\star\star\star$$

We want to place two bars between these stars, so as to divide them into three groups. For instance, we can do:

$$\star\star|\star|\star\star\star\star$$

In this case, Group 1 contains two stars, Group 2 one star, and Group 3 four stars. We are also allowed to have a group that contains no stars, e.g., the following example places all seven stars in Group 3:

$$||\star\star\star\star\star\star\star$$

How many ways are there to divide seven stars into three groups?

b. How many solutions does

$$x_0 + x_1 + x_2 = 10$$

have, if all x's must be *positive* integers?