CS 70 Discrete Mathematics and Probability Theory
Spring 2016 Walrand and Rao

Discussion 12b

1. **Covariance**

   We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $Cov(X_1, X_2)$?

2. **LLSE**

   We have two bags of balls. The fractions of red balls and blue balls in bag A are $\frac{2}{3}$ and $\frac{1}{3}$ respectively. The fractions of red balls and blue balls in bag B are $\frac{1}{2}$ and $\frac{1}{2}$ respectively. Someone gives you one of the bags (unmarked) uniformly at random. Then we draw 6 balls from the same bag with replacement. Let X_i be the indicator random variable that ball i is red. Now, let us define $X = \sum_{1 \le i \le 3} X_i$ and $Y = \sum_{4 \le i \le 6} X_i$. Find $LLSE(Y|X)$. [Hint: recall that $LLSE(Y|X) = E(Y) + \frac{Cov(X,Y)}{Var(X)}(X - E(X))$]

3. **Confidence interval**

   Let $\{X_i\}_{1 \le i \le n}$ be a sequence of iid Bernoulli random variables with parameter μ. Assume we have enough samples such that $P(|\frac{1}{n}\sum_{1 \le i \le n} X_i - \mu| > 0.1) = 0.05$.

   Can you give 95% confidence interval for μ if you are given the outcomes of X_i?