1. Power Inequality

Use induction to prove that for all integers $n \geq 1$, $2^n + 3^n \leq 5^n$.

2. Triangle Inequality

Recall the triangle inequality, which states that for real numbers x_1 and x_2,

$$|x_1 + x_2| \leq |x_1| + |x_2|.$$

Use induction to prove the generalized triangle inequality:

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|.$$

3. (Induction) Prove that, for any positive integer n, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

4. Convergence of Series

Use induction to prove that for all integers $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{3k^{3/2}} \leq 2.$$

Hint: Strengthen the induction hypothesis to $\sum_{k=1}^{n} \frac{1}{3k^{3/2}} \leq 2 - \frac{1}{\sqrt{n}}$.

5. Fibonacci

Recall, the Fibonacci numbers, defined recursively as $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-2} + F_{n-1}$. Prove that every third Fibonacci number is even. For example, $F_3 = 2$ is even and $F_6 = 8$ is even.