1. **Tournament**

A tournament is defined to be a directed graph such that for every pair of distinct nodes \( v \) and \( w \), exactly one of \( (v, w) \) and \( (w, v) \) is an edge (representing which player beat the other in a round-robin tournament). Prove that every tournament has a Hamiltonian path. In other words, you can always arrange the players in a line so that each player beats the next player in the line.

2. **Leaves in a tree**

A leaf in a tree is a vertex with degree 1.

   (a) Prove that every tree on \( n \geq 2 \) vertices has at least two leaves.

   (b) What is the maximum number of leaves in a tree with \( n \geq 3 \) vertices?
3. **Edge-disjoint paths in hypercube**

Prove that between any two distinct vertices \( x, y \) in the \( n \)-dimensional hypercube graph, there are at least \( n \) edge-disjoint paths from \( x \) to \( y \) (i.e., no two paths share an edge, though they may share vertices).

4. **Planarity**

Consider graphs with the property \( T \): For every three distinct vertices \( v_1, v_2, v_3 \) of graph \( G \), there are at least two edges among them. Prove that if \( G \) is a graph on \( \geq 7 \) vertices, and \( G \) has property \( T \), then \( G \) is nonplanar.

5. **Graph Coloring**

Prove that a graph with maximum degree at most \( k \) is \((k + 1)\)-colorable.