1. **Repeated Squaring** Compute $3^{383} \pmod{7}$. (Via repeated squaring!)

2. **Modular Potpourri**
   
   (a) Evaluate $4^{96} \pmod{5}$
   
   (b) Prove or Disprove: There exists some $x \in \mathbb{Z}$ such that $x \equiv 3 \pmod{16}$ and $x \equiv 4 \pmod{6}$.
   
   (c) Prove or Disprove: $2x \equiv 4 \pmod{12} \iff x \equiv 2 \pmod{12}$

3. **Just a Little Proof**
   
   Suppose that $p$ and $q$ are distinct odd primes and $a$ is an integer such that $\gcd(a, pq) = 1$. Prove that $a^{(p-1)(q-1)+1} \equiv a \pmod{pq}$.

4. **Euler’s totient function**
   
   Euler’s totient function is defined as follows:
   
   $$\phi(n) = |\{i : 1 \leq i \leq n, \gcd(n, i) = 1\}|$$
   
   In other words, $\phi(n)$ is the total number of positive integers less than $n$ which are relatively prime to it. Here is a property of Euler’s totient function that you can use without proof:
   
   For $m, n$ such that $\gcd(m, n) = 1$, $\phi(mn) = \phi(m) \cdot \phi(n)$.

   (a) Let $p$ be a prime number. What is $\phi(p)$?
   
   (b) Let $p$ be a prime number and $k$ be some positive integer. What is $\phi(p^k)$?
   
   (c) Let $p$ be a prime number and $a$ be a positive integer smaller than $p$. What is $a^{\phi(p)} \pmod{p}$?
     
     *(Hint: use Fermat’s Little Theorem.)*
   
   (d) Let $b$ be a number whose prime factors are $p_1, p_2, \ldots, p_k$. We can write $b = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots \cdot p_k^{a_k}$.
     
     Show that for any $a$ relatively prime to $b$, the following holds:
     
     $$\forall i \in \{1, 2, \ldots, k\}, \quad a^{\phi(b)} \equiv 1 \pmod{p_i}$$