

### 1. Clothes and stuff

- (a) Say we've decided to do the whole capsule wardrobe thing and we now have only 5 different items of clothing that we wear (jeans, tees, shoes, jackets, and floppy hats, obvi). We have 3 variations on each of the items, and we wear one of each item every day. How many different outfits can we make?
- (b) It turns out 3 floppy hats really isn't enough of a selection, so we've bought 11 more, and we now have 14 floppy hats. Now how many outfits can we make?
- (c) If we own  $k$  different items of clothing, with  $n_1$  variations of the first item,  $n_2$  variations of the second,  $n_3$  of the third, and so on, how many outfits can we make?
- (d) We love our floppy hats so much that we've decided to also use them as wall art, so we're picking 4 of our 14 hats to hang in a row on the wall. How many such arrangements could we make? (Order matters, because no one really wants to see that burgundy one next to our favorite forest green fedora.)
- (e) Ok, now we're packing for vacation to Iceland, and we only have space for 4 of our 14 floppy hats. How many sets of 4 could we bring? (Yeah, yeah, we knew you were going to use that notation. Now tell us the number as a function of  $a$ , your answer from the previous part.)
- (f) Ok, turns out the check-in person for our flight to Iceland is being *very* unreasonable about the luggage weight restrictions, and we're going to have to leave some hats behind. Despite our best intentions, and having packed only 4 hats, we actually bought 18 additional floppy hats at the airport (6 in burgundy, 6 in forest green, and 6 in classic black). We'll keep our 4 hats that we brought from home, but we'll have to return all but 6 of the airport hats. How many color configurations can there be for the 6 airport hats that we keep?

### 2. Counting practice

Leave your answers as (tidy) expressions involving factorials, binomial coefficients, etc., rather than evaluating them as decimal numbers (though you are welcome to perform this last step as well for your own interest if you like, provided it is clearly separated from your main answer). Explain clearly how you arrived at your answers.

- (a) How many 75-bit strings are there that contain more ones than zeros?
- (b) How many bridge hands contain exactly 4 hearts?
- (c) How many anagrams of OKLAHOMA are there? (An anagram of OKLAHOMA is any re-ordering of the letters of OKLAHOMA; i.e., any string made up of the eight letters O, K, L, A, H, O, M and A, in any order. The anagram does not have to be an English word.)
- (d) How many different anagrams of MISSISSIPPI are there?
- (e) How many rolls of 6 dice with at most 5 distinct values?

(f) How many increasing sequences of  $k$  numbers from  $\{1, \dots, n\}$ ? ( $2, 3, 5, 5, 7$  is not an increasing sequence, but  $2, 3, 5, 7$  is.)

(g) How many distinct degree  $\leq d$  polynomials modulo  $p$ , where  $d \leq p - 1$ ?

(h) How many distinct degree  $\leq d$  polynomials with exactly  $d$  roots modulo  $p$  where  $d \leq p - 1$ ?  
(Brief explanation ok.)

(i) How many values of  $x \in \{0, \dots, 11\}$  satisfy  $4x = 6 \pmod{12}$ ?

(j) How many values of  $x \in \{0, \dots, 35\}$  satisfy  $8x = 4 \pmod{36}$ ?