CS 70 Discrete Mathematics and Probability Theory
Spring 2016 Rao and Walrand Discussion 8B

1. **Monty Hall Again**

In the three-door Monty Hall problem, there are two stages to the decision, the initial pick followed by the decision to stick with it or switch to the only other remaining alternative after the host has shown an incorrect door. An extension of the basic problem to multiple stages goes as follow.
Suppose there are four doors, one of which is a winner. The host says: "You point to one of the doors, and then I will open one of the other non-winners. Then you decide whether to stick with your original pick or switch to one of the remaining doors. Then I will open another (other than the current pick) non-winner. You will then make your nal decision by sticking with the door picked on the previous decision or by switching to the only other remaining door.

(a) How many possible strategies are there?

(b) Find the best strategy and compute its probability of winning. You can do this using any method you want. Enumeration is a valid approach, but a less tedious method is to consider a simpler problem: what if you have a 3-door monty hall problem, except the probability of picking the right door at first is not $\frac{1}{3}$ but some general value p? What is the best strategy if $p = 0$? What is the best strategy if $p = 1$? Having done that, how would you reduce the 4-door problem to the above 3-door problem?

2. **Drunk man**

Imagine that you have a drunk man moving along the horizontal axis (that stretches from $x = -\infty$ to $x = +\infty$). At time $t = 0$, his position on this axis is $x = 0$. At each time point $t = 1$, $t = 2$, etc., the man moves forward (that is, $x(t+1) = x(t)+1$) with probability 0.5, backward (that is, $x(t+1) = x(t)-1$) with probability 0.3, and stays exactly where he is (that is, $x(t+1) = x(t)$) with probability 0.2.

(a) What are all his possible positions at time t, $t \geq 0$?

(b) Calculate the probability of each possible position at $t = 1$.

(c) Calculate the probability of each possible position at $t = 2$.

(d) Calculate the probability of each possible position at $t = 3$.

(e) If you know the probability of each position at time t, how will you nd the probabilities at time $t+1$?

The Drunk Man has regained some control over his movement, and no longer stays in the same spot; he only moves forwards or backwards. More formally, let the Drunk Man's initial position be $x(0) = 0$. Every second, he either moves forward one pace or backwards one pace, *i.e.*, his position at time $t+1$ will be one of $x(t+1) = x(t)+1$ or $x(t+1) = x(t)-1$.

We want to compute the number of paths in which the Drunk Man returns to 0 at time t and it is his first return, *i.e.*, $x(t) = 0$ and $x(s) \neq 0$ for all s where $0 < s < t$. Note, we **no longer** care about probabilities. We are just counting paths here.

(a) How many paths can the Drunk Man take if he returns to 0 at $t = 6$ and it is his first return?

(b) How many paths can the Drunk Man take if he returns to 0 at $t = 7$ and it is his first return?

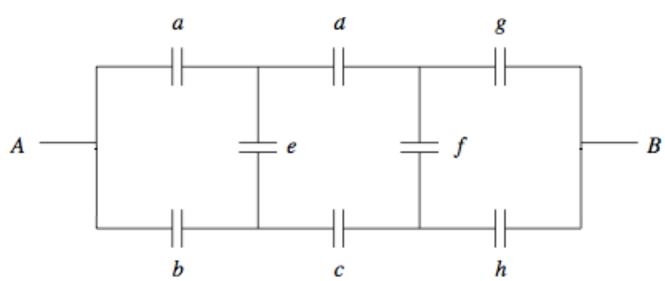
(c) How many paths can the Drunk Man take if he returns to 0 at $t = 8$ and it is his first return?

(d) How many paths can the Drunk Man take if he returns to 0 at $t = 2n+1$ for $n \in \mathbb{N}$ and it is his first return?

(e) How many paths can the Drunk Man take if he returns to 0 at $t = 2n+2$ for $n \in \mathbb{N}$ and it is his first return?

3. **Communication network**

In the communication network shown below, link failures are independent, and each link has a probability of failure of p. Consider the physical situation before you write anything. A can communicate with B as long as they are connected by at least one path which contains only in-service links.



(a) Given that exactly 5 links have failed, determine the probability that A can still communicate with B.

(b) Given that exactly 5 links have failed, determine the probability that either g or h (*but not both*) is still operating properly.

(c) Given that *a, d* and h have failed (but no information about the information of the other links), determine the probability that A can communicate with B.