1. **A roulette of apples** You bought 20 apples at your local farmer’s market. Due to the organic nature of the apples, they are infested with worms. In particular, each apple contains a single worm with probability 0.3 (mutually independent). *(leave answers unevaluated)*

a) You pick up two apples $a_1$ and $a_2$.
   
i) What is the probability that there are exactly 2 worms? What is the probability that there is exactly 1 worm? 0 worms?
   
ii) What is $P(a_1 \text{ has worm} | a_2 \text{ has worm})$?
   
iii) What is $P(a_1 \text{ has worm} | \text{there is exactly 1 worm among } a_1 \text{ and } a_2)$?

b) You eat all 20 apples.
   
i) What is the probability that you end up eating no worms?
   
ii) what is the probability that you end up eating exactly 1 worm?
   
iii) what is the probability that you end up eating exactly 2 worms?
   
iv) How many apples can you eat if you want the probability of eating no worms to be at least 0.2?

c) You pick a single apple at random and slice it into 3 slices. If the apple has a worm, it will be hidden in one of the slices. You bravely eat the slices one by one. Let $s_1$, $s_2$, $s_3$ denote the three slices.
   
i) What is $P(s_1 \text{ has no worm} | \text{apple has worm})$?
   
ii) What is $P(s_1 \text{ has no worm})$?
   
iii) What is $P(\text{apple has no worm} | s_1 \text{ has no worm})$. Compare your answer to $P(\text{apple has no worm})$.
   
iv) What is $P(s_2 \text{ has no worm} | s_1 \text{ has no worm})$ Compare your answer to $P(s_2 \text{ has no worm})$.
   
v) Using your previous answer, what is the safest way to eat 2 slices of apples?
2. Independence in balls and bins

You have $k$ balls and $n$ bins labelled $1, 2, \ldots, n$, where $n \geq 2$. You drop each ball uniformly at random into the bins.

a. What is the probability that bin $n$ is empty?
b. What is the probability that bin 1 is non-empty?
c. What is the probability that both bin 1 and bin $n$ are empty?
d. What is the probability that bin 1 is non-empty and bin $n$ is empty?
e. What is the probability that bin 1 is non-empty given that bin $n$ is empty?
f. What does this tell us about the independence of the two events, $A$: bin 1 is non-empty and $B$: bin $n$ is non-empty?

3. Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in CS70, you are curious to play around with these numbers. Find the probability that

a. A given day is windy and rainy.
b. It rains on a given day.
c. Exactly one of two days is rainy. (Assume that the two days are independent.)
d. A non-rainy day is also non-windy.