

- 1. A roulette of apples** You bought 20 apples at your local farmer's market. Due to the organic nature of the apples, they are infested with worms. In particular, each apple contains a single worm with probability 0.3 (mutually independent). (**leave answers unevaluated**)
- a) You pick up two apples  $a_1$  and  $a_2$ .
- What is the probability that there are exactly 2 worms? What is the probability that there is exactly 1 worm? 0 worms?
  - What is  $P(a_1 \text{ has worm} | a_2 \text{ has worm})$ ?
  - What is  $P(a_1 \text{ has worm} | \text{there is exactly 1 worm among } a_1 \text{ and } a_2)$ ?
- b) You eat all 20 apples.
- What is the probability that you end up eating no worms?
  - what is the probability that you end up eating exactly 1 worm?
  - what is the probability that you end up eating exactly 2 worms?
  - How many apples can you eat if you want the probability of eating no worms to be at least 0.2?
- c) You pick a single apple at random and slice it into 3 slices. If the apple has a worm, it will be hidden in one of the slices. You bravely eat the slices one by one. Let  $s_1, s_2, s_3$  denote the three slices.
- What is  $P(s_1 \text{ has no worm} | \text{apple has worm})$
  - What is  $P(s_1 \text{ has no worm})$ ?
  - What is  $P(\text{apple has no worm} | s_1 \text{ has no worm})$ . Compare your answer to  $P(\text{apple has no worm})$ .
  - What is  $P(s_2 \text{ has no worm} | s_1 \text{ has no worm})$  Compare your answer to  $P(s_2 \text{ has no worm})$ .
  - Using your previous answer, what is the safest way to eat 2 slices of apples?

## 2. Independence in balls and bins

You have  $k$  balls and  $n$  bins labelled  $1, 2, \dots, n$ , where  $n \geq 2$ . You drop each ball uniformly at random into the bins.

- a. What is the probability that bin  $n$  is empty?
- b. What is the probability that bin 1 is non-empty?
- c. What is the probability that both bin 1 and bin  $n$  are empty?
- d. What is the probability that bin 1 is non-empty and bin  $n$  is empty?
- e. What is the probability that bin 1 is non-empty given that bin  $n$  is empty?
- f. What does this tell us about the independence of the two events,  $A$ : bin 1 is non-empty and  $B$ : bin  $n$  is non-empty?

## 3. Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in CS70, you are curious to play around with these numbers. Find the probability that

- a. A given day is windy and rainy.
- b. It rains on a given day.
- c. Exactly one of two days is rainy. (Assume that the two days are independent.)
- d. A non-rainy day is also non-windy.