Programming Computers
70: Discrete Math and Probability.

Programming Computers ≡ Superpower!
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What are your super powerful programs doing?
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What are your super powerful programs doing?
Logic and Proofs!
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
  Logic and Proofs!
  Induction ≡ Recursion.
Programming Computers $\equiv$ Superpower!

What are your super powerful programs doing?
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What can computers do?
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What can computers do?
   Work with discrete objects.
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   Discrete Math
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   **Discrete Math** $\implies$ immense application.
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Computers learn and interact with the world?
70: Discrete Math and Probability.

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Computers learn and interact with the world?
   E.g. machine learning, data analysis.
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  Discrete Math ⇒ immense application.

Computers learn and interact with the world?
  E.g. machine learning, data analysis.
  Probability!

See note 1, for more discussion.
Admin.

Course Webpage: inst.cs.berkeley.edu/~cs70/sp16
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Explains policies, has homework, midterm dates, etc.
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Two midterms, final.
Admin.

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Two midterms, final.
midterm 1 before drop date. (2/16)
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Assessment: Two options:

Test Only.
   Midterm 1: 25%
   Midterm 2: 25%
   Final: 49%
   Sundry: 1%
Admin.

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Test Only.
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  Midterm 2: 25%
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  Sundry: 1%

Test plus Homework.
  Test Only Score: 85%
  Homework Score: 15%
Instructors: Satish Rao and Jean Walrand.
Instructors: Satish Rao and Jean Walrand. Both are available throughout the course.
Instructor/Admin

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Satish Rao: mostly discrete math.
Jean Walrand: mostly probability.
Jean Walrand – Prof. of EECS – UCB
257 Cory Hall – walrand@berkeley.edu

I was born in Belgium\(^{(1)}\) and came to Berkeley for my PhD. I have been teaching at UCB since 1982.

My wife and I live in Berkeley. We have two daughters (UC alumni – Go Bears!). We like to ski and play tennis (both poorly). We enjoy classical music and jazz.

My research interests include stochastic systems, networks and game theory.

\(^{(1)}\)
17th year at Berkeley.
Satish Rao

17th year at Berkeley.
PhD: Long time ago,
17th year at Berkeley.
PhD: Long time ago, far
17th year at Berkeley.
PhD: Long time ago, far far away.
Satish Rao

17th year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory
Satish Rao

17th year at Berkeley.
PhD: Long time ago, far far away.
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17th year at Berkeley.
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Taught: 170, 174, 70, 270, 273, 294,
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Satish Rao

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Recovering Helicopter(ish) parent of 3 College(ish) kids.
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Wason’s experiment:1

Suppose we have four cards on a table:

1. 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
Wason’s experiment: 1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
Wason’s experiment:1

Suppose we have four cards on a table:

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- Consider the theory:
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory:
  “If a person travels to Chicago, he/she flies.”
Wason’s experiment:1

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▶ 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
▶ Card contains person’s destination on one side, and mode of travel.
▶ Consider the theory: “If a person travels to Chicago, he/she flies.”
▶ Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.
Wason’s experiment:1

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Which cards do you need to flip to test the theory?
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Answer:
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Which cards do you need to flip to test the theory?

Answer: Later.
CS70: Lecture 1. Outline.

Today: Note 1.
Today: Note 1. Note 0 is background.
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The language of proofs!
CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background. Do read/skim it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
\[ 2 + 2 = 3 \]
\[ 826 \text{th digit of } \pi \text{ is } 4 \]
\[ \text{Johny Depp is a good actor} \]
\[ \text{All evens } > 2 \text{ are sums of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
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Johny Depp is a good actor \quad \text{Not a Proposition}

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\[
\begin{array}{l|l|l}
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Again: “value” of a proposition is ...
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Again: “value” of a proposition is ... **True or False**
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True when both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

"2 + 2 = 4" ∧ "2 + 2 = 3" – a proposition that is False.

"2 + 2 = 3" ∨ "2 + 2 = 4" – a proposition that is True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

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"$P \lor Q$" is True when at least one $P$ or $Q$ is True.
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Negation ("not"): $\neg P$
Propositional Forms.

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Examples:
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is **True** when both \( P \) and \( Q \) are **True**. Else **False**.

Disjunction (“or”): \( P \lor Q \)

“\( P \lor Q \)” is **True** when at least one \( P \) or \( Q \) is **True**. Else **False**.

Negation (“not”): \( \neg P \)

“\( \neg P \)” is **True** when \( P \) is **False**. Else **False**.

Examples:

\( \neg "(2 + 2 = 4)" \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.

Disjunction ("or"): \( P \lor Q \)

"\( P \lor Q \)" is True when at least one \( P \) or \( Q \) is True. Else False.

Negation ("not"): \( \neg P \)

"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg \) "\( (2 + 2 = 4) \)" – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.

Disjunction ("or"): \( P \lor Q \)

"\( P \lor Q \)" is True when at least one \( P \) or \( Q \) is True. Else False.

Negation ("not"): \( \neg P \)

"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg "(2 + 2 = 4)" \) – a proposition that is ... False

"2 + 2 = 3" \( \land "2 + 2 = 4" \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True when at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3" \land "2 + 2 = 4$" – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

\( P \land Q \) is True when both \( P \) and \( Q \) are True. Else False.

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\( \neg P \) is True when \( P \) is False. Else False.

Examples:

\( \neg \ "(2 + 2 = 4)" \) – a proposition that is ... False

\( 2 + 2 = 3 \) \( \land \) \( 2 + 2 = 4 \) – a proposition that is ... False

\( 2 + 2 = 3 \) \( \lor \) \( 2 + 2 = 4 \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

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"$2 + 2 = 3" \land "2 + 2 = 4$" – a proposition that is ... False

"$2 + 2 = 3" \lor "2 + 2 = 4$" – a proposition that is ... True
Propositional Forms.

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Examples:

\( \neg "(2 + 2 = 4)" \) — a proposition that is ... False

“2 + 2 = 3” \( \land \) “2 + 2 = 4” — a proposition that is ... False

“2 + 2 = 3” \( \lor \) “2 + 2 = 4” — a proposition that is ... True
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]
Propositional Forms: quick check!

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Propositional Forms: quick check!

\( P = \text{“} \sqrt{2} \text{ is rational”} \)
\( Q = \text{“} 826\text{th digit of pi is 2”} \)

\( P \) is ...
Propositional Forms: quick check!

\[ P = \text{"\(\sqrt{2}\) is rational"} \]
\[ Q = \text{"826th digit of pi is 2"} \]

\[ P \text{ is ...} \text{False .} \]
Propositional Forms: quick check!

$P = \text{“} \sqrt{2} \text{ is rational”}$
$Q = \text{“} 826\text{th digit of pi is 2”}$

$P$ is ... True.
$Q$ is ... False.
Propositional Forms: quick check!

\[ P = \text{"sqrt\(2\) is rational"} \]
\[ Q = \text{"826th digit of pi is 2"} \]

\[ P \text{ is ... } \text{False} . \]
\[ Q \text{ is ... } \text{True} . \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]

\[ P \] is ... False .
\[ Q \] is ... True .

\[ P \land Q \] ...
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational} \]  
\[ Q = \text{“} 826\text{th digit of } \pi \text{ is 2} \]  

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Propositional Forms: quick check!

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\( P = \text{“} \sqrt{2} \text{ is rational”} \)
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\( P \) is ... **False**.
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\( P \land Q \) ... False
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\( \neg P \) ... True
Put them together.

**Propositions:**

$P_1$ - Person 1 rides the bus.
Put them together..

Propositions:

- $P_1$ - Person 1 rides the bus.
- $P_2$ - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn’t.

Propositional Form:

$$\neg ((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))$$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?
This seems complicated.
We can program!!!!
We need a way to keep track!
Put them together..

Propositions:

\(P_1\) - Person 1 rides the bus.
\(P_2\) - Person 2 rides the bus.
....
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Notice: $\land$ and $\lor$ are commutative.
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DeMorgan's Law's for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

$\neg (P \lor Q) \equiv \neg P \land \neg Q$
Truth Tables for Propositional Forms.

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**DeMorgan's Law's for Negation:**
- $\neg(P \land Q) \equiv \neg P \lor \neg Q$
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Notice: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

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One use for truth tables: Logical Equivalence of propositional forms!

Example:

$\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$...

DeMorgan's Law's for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

$\neg (P \lor Q) \equiv \neg P \land \neg Q$
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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$
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Example: $\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$

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Notice: ∧ and ∨ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: \( \neg(P ∧ Q) \) logically equivalent to \( \neg P ∨ \neg Q \)
...because the two propositional forms have the same...
...Truth Table!

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Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$
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DeMorgan’s Law’s for Negation: distribute and flip!
$\neg(P \land Q)$
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$\neg(P \land Q) \equiv \neg P \lor \neg Q$
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DeMorgan’s Law’s for Negation: distribute and flip!

\[ \neg(P \land Q) \equiv \neg P \lor \neg Q \]
\[ \neg(P \lor Q) \equiv \neg P \land \neg Q \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q , \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:

- \(P\) is \textbf{True}.
  - LHS: \(T \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\( P \) is \textbf{True} .

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \, ? \]

Simplify: \((T \land Q) \equiv Q\), \((F \land Q) \equiv F\).

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).

RHS: \((T \land Q) \lor (T \land R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

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\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:

- \(P\) is True .
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

- \(P\) is False .
Distributive?

\( P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \)

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\( P \) is True .

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) . \)

RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) . \)

\( P \) is False .

LHS: \( F \land (Q \lor R) \)
Distributive?

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Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

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Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

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Distributive?

\( P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \)?

Simplify: \( T \land Q \equiv Q \), \( F \land Q \equiv F \).

Cases:

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LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).

RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).

\( P \) is False.

LHS: \( F \land (Q \lor R) \equiv F \).

RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\( P \) is True.
- LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
- RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).

\( P \) is False.
- LHS: \( F \land (Q \lor R) \equiv F \).
- RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F \).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]?

Simplify: \( T \land Q \equiv Q \), \( F \land Q \equiv F \).

Cases:

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LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).

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\( P \) is False .

LHS: \( F \land (Q \lor R) \equiv F \).

RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F \).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, \ (F \land Q) \equiv F. \)

Cases:

- **P** is **True** .
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

- **P** is **False** .
  - LHS: \( F \land (Q \lor R) \equiv F. \)
  - RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

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LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

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RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T,\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

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LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is \textbf{False}.

LHS: \(F \land (Q \lor R) \equiv F.\)
RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q.\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:

- **P is True**.
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

- **P is False**.
  - LHS: \( F \land (Q \lor R) \equiv F. \)
  - RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q. \)

Foil 1:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:

\( P \) is True.

LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)

RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

\( P \) is False.

LHS: \( F \land (Q \lor R) \equiv F. \)

RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q. \)

Foil 1:

\[ (A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\( P \) is True .

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\). 

\( P \) is False .

LHS: \( F \land (Q \lor R) \equiv F \).

RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q \).

Foil 1:

\((A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D) ? \)

Foil 2:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
- **P is True** .
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
- **P is False** .
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q.\)

Foil 1:
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(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)?
\]

Foil 2:
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(A \land B) \lor (C \land D) \equiv (A \lor C) \land (A \lor D) \land (B \lor C) \land (B \lor D)?
\]
Implication.

\[ P \implies Q \text{ interpreted as} \]

\[ P \implies Q \]
Implication.

\[ P \implies Q \text{ interpreted as} \]

If \( P \), then \( Q \).
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\[ P \implies Q \text{ interpreted as} \]

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).

Examples:

Statement: If you stand in the rain, then you'll get wet.

\( P = \) "you stand in the rain"

\( Q = \) "you will get wet."

Statement: "Stand in the rain"

Can conclude: "you'll get wet."

Statement: If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).

\( P = \) "a right triangle has sidelengths \( a \leq b \leq c \),"

\( Q = \) "\( a^2 + b^2 = c^2 \)."
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.
Implication.

\[ P \implies Q \text{ interpreted as } \]

If \( P \), then \( Q \).

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Non-Consequences/consequences of Implication

The statement \( P \implies Q \) only is False if \( P \) is True and \( Q \) is False. False implies nothing. \( P \) False means \( Q \) can be True or False. Anything implies true. \( P \) can be True or False when \( Q \) is True. If chemical plant pollutes river, fish die. If fish die, did chemical plant pollute river? Not necessarily. \( P \implies Q \) and \( Q \) are True does not mean \( P \) is True. Be careful! Instead we have: \( P \implies Q \) and \( P \) are True does mean \( Q \) is True. The chemical plant pollutes river. Can we conclude fish die? Some Fun: use propositional formulas to describe implication? \(( ( P \implies Q ) \land P ) \implies Q \).
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The statement “$P \implies Q$” only is False if $P$ is True and $Q$ is False.
Non-Consequences/Consequences of Implication

The statement "\( P \implies Q \)"

only is **False** if \( P \) is **True** and \( Q \) is **False**.

False implies nothing
Non-Consequences/consequences of Implication

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Non-Consequences/consequences of Implication

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If chemical plant pollutes river, fish die.
Non-Consequences/consequences of Implication

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Some Fun: use propositional formulas to describe implication?
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Some Fun: use propositional formulas to describe implication?
\(((P \implies Q) \land P) \implies Q\).
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).

Just reversing the order.

\[ P \iff Q \]

- \( P \) only if \( Q \).

Remember if \( P \) is true then \( Q \) must be true. This suggests that \( P \) can only be true if \( Q \) is true. Since if \( Q \) is false \( P \) must have been false.

\[ P \] is sufficient for \( Q \).

This means that proving \( P \) allows you to conclude that \( Q \) is true.

\[ Q \] is necessary for \( P \).

For \( P \) to be true it is necessary that \( Q \) is true. Or if \( Q \) is false then we know that \( P \) is false.
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Implication and English.

\[ P \Rightarrow Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  - Just reversing the order.
- \( P \) only if \( Q \).
  - Remember if \( P \) is true then \( Q \) must be true.
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Truth Table: implication.

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These two propositional forms are logically equivalent!
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$\neg P \lor Q \equiv P \implies Q$. 

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$\neg P \lor Q \equiv P \implies Q$.

These two propositional forms are logically equivalent!
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$. 
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

- Converse of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.

- Not logically equivalent!
Contrapositive, Converse

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Contrapositive, Converse

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  - If the plant pollutes, fish die.
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    (contrapositive)
Contrapositive, Converse

- Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).
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- Converse of $P \implies Q$ is $Q \implies P$.
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- Not logically equivalent!

- Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$.
  (Logically Equivalent: $\iff$.)
Contrapositive, Converse

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Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q$
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Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P$
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- Converse of $P \implies Q$ is $Q \implies P$.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or
  $P \iff Q$.
  (Logically Equivalent: $\iff$. )
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \[ x > 2 \]
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \( x > 2 \)
- \( n \) is even and the sum of two primes
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
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No. They have a free variable.
Variables.

Propositions?

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We call them predicates, e.g., \( Q(x) = "x \text{ is even}" \)
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Same as boolean valued functions from 61A or 61AS!
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- \( P(n) = "\sum_{i=1}^{n} i = \frac{n(n+1)}{2}." \)
- \( R(x) = "x > 2" \)

Theory from Wason's.

If person \( x \) goes to Chicago then person \( x \) flew.

Next:

Statements about boolean valued functions!!
Variables.
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
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Variables.

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\[ G(n) = \text{“} n \text{ is even and the sum of two primes”} \]

Remember Wason’s experiment!

\[ F(x) = \text{“} \text{Person} \ x \text{ flew.”} \]
Variables.
Propositions?

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- $C(x) \implies F(x)$. 
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Propositions?

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- $C(x) \implies F(x)$. Theory from Wason’s.
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\[ C(x) \implies F(x). \text{ Theory from Wason’s.} \]

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Next:
Variables.

Propositions?

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- \( C(x) \implies F(x) \). Theory from Wason’s.
  If person \( x \) goes to Chicago then person \( x \) flew.

Next: Statements about boolean valued functions!!
Quantifiers..

There exists quantifier:

$$\exists x \in S \ (P(x))$$

means "There exists an $x$ in $S$ where $P(x)$ is true."

For example:

$$\exists x \in \mathbb{N} \ (x = x^2)$$

Equivalent to "

$$0 = 0 \lor 1 = 1 \lor 2 = 4 \lor \ldots$$"

Much shorter to use a quantifier!

For all quantifier;

$$\forall x \in S \ (P(x))$$

means "For all $x$ in $S$, we have $P(x)$ is True."

Examples:

"Adding 1 makes a bigger number."

$$\forall x \in \mathbb{N} \ (x + 1 > x)$$

"the square of a number is always non-negative"

$$\forall x \in \mathbb{N} \ (x^2 \geq 0)$$

Wait!

What is $\mathbb{N}$?
There exists quantifier:

\((\exists x \in S)(P(x))\) means "There exists an \(x\) in \(S\) where \(P(x)\) is true."

For example:

\((\exists x \in \mathbb{N})(x^2 = x)\)

Equivalent to "\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)"

Much shorter to use a quantifier!

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\((\forall x \in S)(P(x))\) means "For all \(x\) in \(S\), we have \(P(x)\) is True."

Examples:

"Adding 1 makes a bigger number."

\((\forall x \in \mathbb{N})(x + 1 > x)\)

"the square of a number is always non-negative"

\((\forall x \in \mathbb{N})(x^2 \geq 0)\)

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For example:

\((\exists x \in \mathbb{N})(x = x^2)\)

Wait! What is \(\mathbb{N}\)?
Quantifiers..

There exists quantifier:

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Equivalent to “$(0 = 0)$”
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Quantifiers..

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Examples:

“Adding 1 makes a bigger number.”
Quantifiers..

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Examples:

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\((\forall x \in \mathbb{N}) (x + 1 > x)\)

"the square of a number is always non-negative"
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Wait! What is \(\mathbb{N}\)?
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe:
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include:

- $\mathbb{N} = \{0, 1, 2, \ldots\}$ (natural numbers)
- $\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ (integers)
- $\mathbb{Z}^+ \text{ (positive integers)}$
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{\text{Alice}, \text{Bob}, \text{Charlie}, \text{Donna}\}$

See note 0 for more!
Quantifiers: universes.

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▶ Any set: $S = \{Alice, Bob, Charlie, Donna\}$. 

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Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

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Theory:

"If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew. Which cards do you need to flip to test the theory?

\[ P(x) = \text{"Person } x \text{ went to Chicago."} \]
\[ Q(x) = \text{"Person } x \text{ flew."} \]

Statement/theory:

\[ \forall x \in \{A, B, C, D\}, P(x) \Rightarrow Q(x) \]

\[ P(A) = \text{False.} \]
Do we care about \( Q(A) \)? No.

\[ P(B) = \text{False.} \]
Do we care about \( P(B) \)? Yes.\[ P(B) = \Rightarrow Q(B), \text{ when } P(B) \text{ is False, } Q(B) \text{ can be anything.} \]

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Do we care about \( P(B) \)? Yes.\[ P(B) = \Rightarrow Q(B), \text{ when } P(B) \text{ is False, } Q(B) = \neg P(B). \]
So \( P(B) \) must be False.

\[ P(C) = \text{True.} \]
Do we care about \( P(C) \)? Yes.\[ P(C) = \Rightarrow Q(C), \text{ means } Q(C) \text{ must be true.} \]

\[ Q(D) = \text{True.} \]
Do we care about \( P(D) \)? No.\[ P(D) = \Rightarrow Q(D) \text{ holds whatever } P(D) \text{ is when } Q(D) \text{ is true.} \]

Only have to turn over cards for Bob and Charlie.
Back to: Wason’s experiment:1

Theory:
“If a person travels to Chicago, he/she flies.”
Back to: Wason’s experiment: 1

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\( Q(B) = \text{False} \). Do we care about \( P(B) \)?
Yes.
Back to: Wason’s experiment:1

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“If a person travels to Chicago, he/she flies.”

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Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).

So \( P(\text{Bob}) \) must be \text{False} \.
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   Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).
   So \( P(\text{Bob}) \) must be \text{False} .

\( P(C) = \text{True} \).
**Back to: Wason’s experiment: 1**

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Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).

So \( P(Bob) \) must be False.

\( P(C) = \text{True} \). Do we care about \( P(C) \)?
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So \( P(\text{Bob}) \) must be False.

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   So \( P(\text{Bob}) \) must be False .

\( P(C) = \text{True} \). Do we care about \( P(C) \)?
   Yes. \( P(C) \implies Q(C) \) means \( Q(C) \) must be true.
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\[ P(x) = \text{“Person } x \text{ went to Chicago.”} \quad Q(x) = \text{“Person } x \text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, P(x) \implies Q(x) \)

\( P(A) = \text{False} \). Do we care about \( Q(A) \)?
No. \( P(A) \implies Q(A) \), when \( P(A) \) is False , \( Q(A) \) can be anything.

\( Q(B) = \text{False} \). Do we care about \( P(B) \)?
Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).
So \( P(\text{Bob}) \) must be False .

\( P(C) = \text{True} \). Do we care about \( P(C) \)?
Yes. \( P(C) \implies Q(C) \) means \( Q(C) \) must be true.

\( Q(D) = \text{True} \).
Back to: Wason’s experiment: 1

Theory:
“If a person travels to Chicago, he/she flies.”

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Only have to turn over cards for Bob and Charlie.
More for all quantifiers examples.

*doubling a number always makes it larger* 
\[
(\forall x \in \mathbb{N}) \quad (2x > x)
\]

False 
Can fix statement...

\[
(\forall x \in \mathbb{N}) \quad (2x \geq x)
\]

True

*Square of any natural number greater than 5 is greater than 25.*
\[
(\forall x \in \mathbb{N}) \quad (x > 5 \implies x^2 > 25)
\]

Idea alert: Restrict domain using implication.
Note that we may omit universe if clear from context.
More for all quantifiers examples.

- “doubling a number always makes it larger”
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- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x) \quad \text{False}\]
More for all quantifiers examples.

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\[(\forall x \in \mathbb{N})(2x > x) \quad \text{False} \quad \text{Consider} \ x = 0\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

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  (\forall x \in N) \ (2x > x) \quad \text{False} \quad \text{Consider } x = 0
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➤ “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in N)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

  \( (\forall x \in N) (2x > x) \)  \textbf{False}  \textbf{Consider} \( x = 0 \)

  Can fix statement...

  \( (\forall x \in N) (2x \geq x) \)  \textbf{True}

- “Square of any natural number greater than 5 is greater than 25.”

  \( (\forall x \in N)(x > 5) \)
More for all quantifiers examples.

- “doubling a number always makes it larger”

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- “Square of any natural number greater than 5 is greater than 25.”

\[ (\forall x \in N) \ (x > 5 \implies \) ]
More for all quantifiers examples.

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Idea alert: Restrict domain using implication.
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Note that we may omit universe if clear from context.
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

- In English: “the square of every natural number is a natural number.”
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In English: “there is a natural number that is the square of every natural number”.

\((\exists y \in N)\)
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) \ (\forall x \in N)\]

\[y = x^2\]
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\]
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\[(\forall x \in \mathbb{N})(\exists y \in \mathbb{N}) (y = x^2)\] True
Consider

\[ \neg (\forall x \in S)(P(x)), \]

What we do in this course! We consider claims.

Claim:

\[ (\forall x) P(x) \]

"For all inputs x the program works."

For False, find \[ x \] where \[ \neg P(x) \].

Counterexample. Bad input. Case that illustrates bug.

For True: prove claim.

Next lectures...
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

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\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]
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Quantifiers....negation...DeMorgan again.

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   - Counterexample.
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For **True** : prove claim.
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- Counterexample.
- Bad input.
- Case that illustrates bug.

For **True**: prove claim. Next lectures...
Negation of exists.

Consider

\[ \neg \left( \exists x \in S \right) \left( P(x) \right) \]

English: means that for all \( x \) in \( S \), \( P(x) \) does not hold.

That is,

\[ \neg \left( \exists x \in S \right) \left( P(x) \right) \iff \forall x \in S \neg P(x) \]
Negation of exists.

Consider

\[ \neg (\exists x \in S)(P(x)) \]
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

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Which Theorem?

Theorem: \(( \forall n \in \mathbb{N} ) \neg ( \exists a, b, c \in \mathbb{N} ) \ ( n \geq 3 \implies a^n + b^n = c^n )\)
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Which Theorem?
Fermat’s Last Theorem!
Which Theorem?

Theorem: $(\forall n \in \mathbb{N}) \neg(\exists a, b, c \in \mathbb{N})(n \geq 3 \implies a^n + b^n = c^n)$

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Fermat’s Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...
Which Theorem?

Theorem: \((\forall n \in N) \neg (\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)\)

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1637: Proof doesn’t fit in the margins.
Theorem: $(\forall n \in \mathbb{N}) \neg(\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)$

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Theorem: \(\neg(\exists n \in \mathbb{N}) (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)\)
Summary.

Propositions are statements that are true or false.
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Propositional forms use $\land, \lor, \neg$. 

DeMorgan's Laws: "Flip and Distribute negation"

$\neg(P \lor Q) \iff \neg P \land \neg Q$

$\neg\forall x P(x) \iff \exists x \neg P(x)$. 

Next Time: proofs!
Summary.

Propositions are statements that are true or false.

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Propositional forms correspond to truth tables.
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Contrapositive: $\neg Q \implies \neg P$
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Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

DeMorgans Laws: "Flip and Distribute negation"

$\neg (P \vee Q) \iff \neg P \wedge \neg Q$

$\neg \forall x P(x) \iff \exists x \neg P(x)$

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Predicates: Statements with “free” variables.
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Implication: $P \implies Q$ $\iff \neg P \lor Q$.

Contrapositive: $\neg Q$ $\implies \neg P$

Converse: $Q$ $\implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x)$, $\exists y \ Q(y)$
Summary.

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DeMorgans Laws: “Flip and Distribute negation”

$\neg(P \lor Q) \iff (\neg P \land \neg Q)$

$\neg\forall x \ P(x) \iff \exists x \ \neg P(x)$
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$\neg(P \lor Q) \iff (\neg P \land \neg Q)$
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Next Time: proofs!