70: Discrete Math and Probability.

Programming Computers

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Programming Computers $\equiv$ Superpower!

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- Logic and Proofs!
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See note 1, for more discussion.

Admin.

Course Webpage: **inst.cs.berkeley.edu/~cs70/sp16**

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Explains policies, has homework, midterm dates, etc.

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Two midterms, final.

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Test Only.
Midterm 1: 25%
Midterm 2: 25%
Final: 49%
Sundry: 1%

Admin.

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Also: Available after class.
Assessment: Two options:

Test Only.
Midterm 1: 25%
Midterm 2: 25%
Final: 49%
Sundry: 1%

Test plus Homework.
Test Only Score: 85%
Homework Score: 15%

Instructor/Admin

Instructors: Satish Rao and Jean Walrand.

Instructor/Admin

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Both are available throughout the course.

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Satish Rao: mostly discrete math.

Jean Walrand: mostly probability.

Jean Walrand – Prof. of EECS – UCB
257 Cory Hall – walrand@berkeley.edu

I was born in Belgium[1] and came to Berkeley for my PhD. I have been teaching at UCB since 1982.

My wife and I live in Berkeley. We have two daughters (UC alumni – Go Bears!). We like to ski and play tennis (both poorly). We enjoy classical music and jazz.

My research interests include stochastic systems, networks and game theory.

(1)

Satish Rao

17th year at Berkeley.

Satish Rao

17th year at Berkeley.
PhD: Long time ago,

Satish Rao

17th year at Berkeley.
PhD: Long time ago, far

Satish Rao

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PhD: Long time ago, far far away.

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17th year at Berkeley.
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Recovering Helicopter(ish) parent of 3 College(ish) kids.

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Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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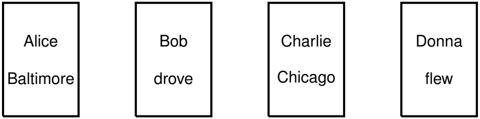
- Consider the theory: "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



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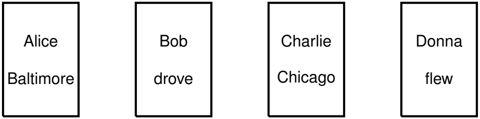

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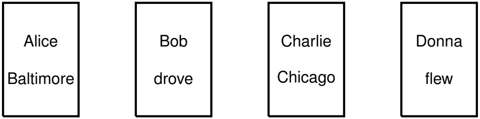

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Answer:

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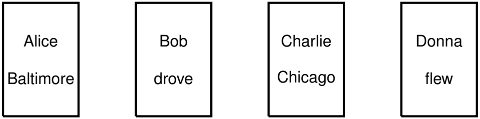

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Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Johny Depp is a good actor
All evens > 2 are sums of 2 primes
$4 + 5$
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Alice travelled to Chicago

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Propositional Forms.

Put propositions together to make another...

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$\neg$ "$(2+2=4)$" – a proposition that is ...

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$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

"$2+2=3$" $\bigvee$ "$2+2=4$" – a proposition that is ... True

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

"$2+2=3$" $\bigvee$ "$2+2=4$" – a proposition that is ... True

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

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P is ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
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P is ...False .

Propositional Forms: quick check!

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P is ...False .
Q is ...

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Propositional Forms: quick check!

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$P \wedge Q$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
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P is ...False .
Q is ...True .

$P \land Q$ ... False

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
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P is ...False .
Q is ...True .

$P \wedge Q$ ... False

$P \vee Q$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... False

$P \vee Q$ ... True

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... False

$P \vee Q$ ... True

$\neg P$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... False

$P \vee Q$ ... True

$\neg P$ ... True

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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Propositions:
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P_2 - Person 2 rides the bus.
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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

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$$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$$

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Propositional Form:
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Can person 3 ride the bus?

Put them together..

Propositions:
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P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
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This seems ...

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This seems ...complicated.

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Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

Notice: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\qquad$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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P is True .

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P is False .

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RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F)$

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.

P is False .

LHS: $F \land (Q \lor R) \equiv F$.

RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:
P is True .
LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.
RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.
P is False .
LHS: $F \land (Q \lor R) \equiv F$.
RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$?

Simplify: $T \lor Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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$P \implies Q$ interpreted as

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Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"

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Statement: If you stand in the rain, then you'll get wet.
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Statement: If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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only is False if P is True and Q is False .

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If chemical plant pollutes river, fish die.

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.

Implication and English.

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Implication and English.

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- If P, then Q.
- Q if P.
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- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.

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- P is sufficient for Q.
  This means that proving P allows you
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  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
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Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
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P	Q	$\neg P \vee Q$
T	T	
T	F	
F	T	
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Truth Table: implication.

P	Q	$P \implies Q$
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$\neg P \vee Q \quad \equiv P \implies Q$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \lor Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \lor Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

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- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
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- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
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  - If you stand in the rain, you get wet.

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  - If you did not stand in the rain, you did not get wet. (not contrapositive!)

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
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  - If you stand in the rain, you get wet.
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  - If you did not get wet, you did not stand in the rain.

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Logically equivalent! Notation: $\equiv$.

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  - If the plant pollutes, fish die.
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$P \implies Q$

Contrapositive, Converse

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Logically equivalent! Notation: $\equiv$.
$P \implies Q \equiv \neg P \vee Q$

Contrapositive, Converse

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  - If the plant pollutes, fish die.
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  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P$

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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Wait! What is $\mathbb{N}$?

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Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

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- See note 0 for more!

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Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- “doubling a number always makes it larger”

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- "Square of any natural number greater than 5 is greater than 25."

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Note that we may omit universe if clear from context.

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- In English: “there is a natural number that is the square of every natural number”.

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For True : prove claim. Next lectures...

Negation of exists.

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Theorem: $(\forall n \in N) \neg(\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

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Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

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1993: Wiles ...(based in part on Ribet's Theorem)

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Next Time: proofs!