Today.

Polynomials.

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Erasure Codes.

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Erasure Codes.

Error Correcting Codes.

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Error Correcting Codes.
  Heads will explode.

Finite Fields

Modular Fact!!!

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Proof works for reals, rationals, and complex numbers.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

Secret Sharing

Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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(Almost) the same as what is missing: one $P(i)$.

Runtime.

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Runtime: polynomial in k, n, and $\log p$.

1. Evaluate degree $k-1$ polynomial n times using $\log p$-bit numbers.
2. Reconstruct secret by solving system of k equations using $\log p$-bit arithmetic.

A bit more counting.

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Infinite number for reals, rationals, complex numbers!

Next

Polynomials and Coding theory.

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

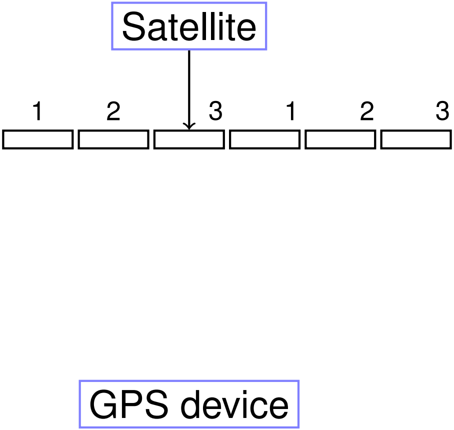
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



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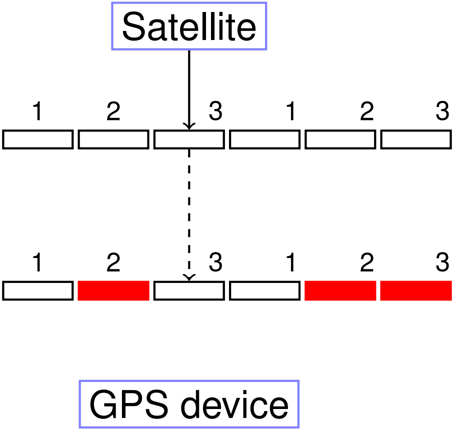

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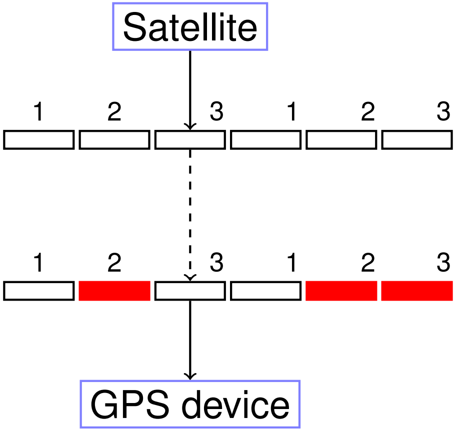

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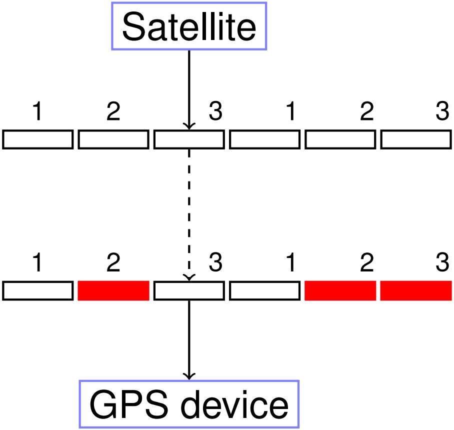

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Use polynomials.

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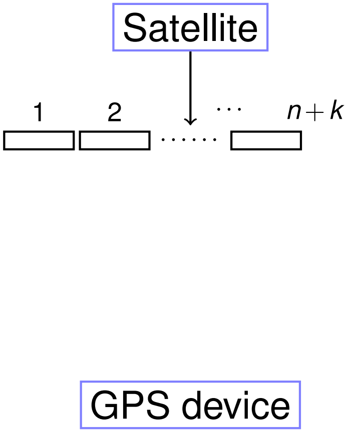

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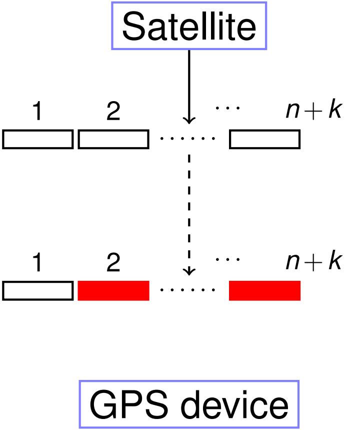

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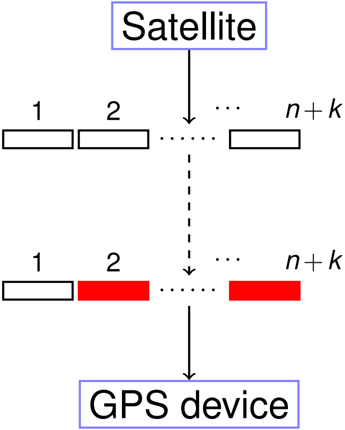

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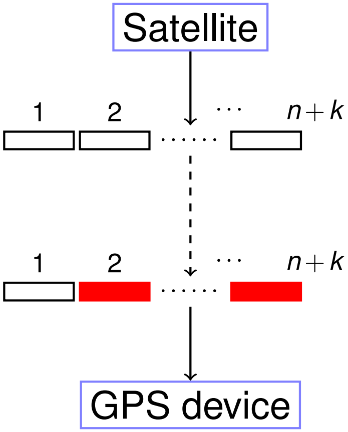

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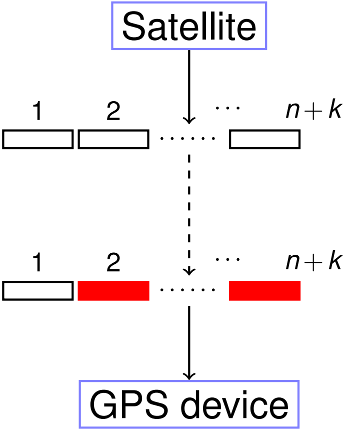

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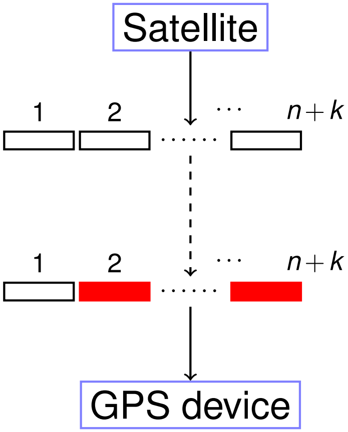

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Make polynomial with $P(1) = 1$, $P(2) = 4$, $P(3) = 4$.

How?

Lagrange Interpolation.
Linear System.

Work modulo 5.

$P(x) = x^2 \pmod 5$
$P(1) = 1, P(2) = 4, P(3) = 9 = 4 \pmod 5$

Send $(0, P(0)) \ldots (5, P(5))$.

6 points.

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Modulo 7 to accommodate at least 6 packets.

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$a_1 = 2a_0$.

Example

Make polynomial with $P(1)=1$, $P(2)=4$, $P(3)=4$.

Modulo 7 to accommodate at least 6 packets.

Linear equations:

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

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Notice that packets contain “x-values”.

Bad reception!

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Channeling Sahai

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Message? $P(1) = 1, P(2) = 4, P(3) = 4$.

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You want to encode a secret consisting of 1,4,4.

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You want to send a message consisting of packets 1,4,2,3,0

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How big should modulus be?

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You want to send a message consisting of packets 1,4,2,3,0

through a noisy channel that loses 3 packets.

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How big should modulus be?
Larger than 8

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Send n packets b-bit packets, with k errors.

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through a noisy channel that loses 3 packets.

How big should modulus be?
Larger than 8 and prime!

Send n packets b-bit packets, with k errors.
Modulus should be larger than $n+k$ and also larger than 2^b.

Polynomials.

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- ..give Secret Sharing.

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- ..give Secret Sharing.
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Error Correction:

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Noisy Channel: corrupts k packets. (rather than loss.)

Polynomials.

- ..give Secret Sharing.
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Error Correction:

Noisy Channel: corrupts k packets. (rather than loss.)

Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

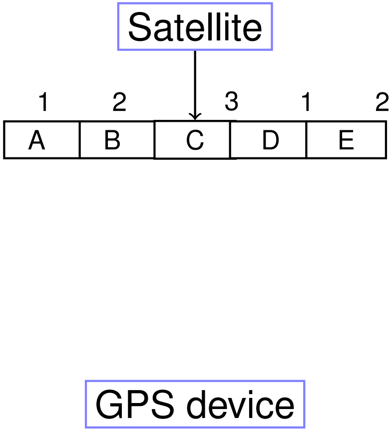
Satellite

3 packet message.

Corrupts 1 packets.

GPS device

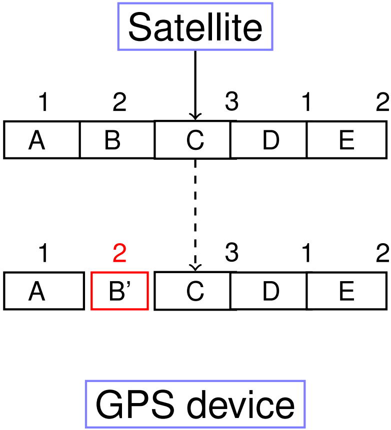
Error Correction



3 packet message. Send 5.

Corrupts 1 packets.

Error Correction



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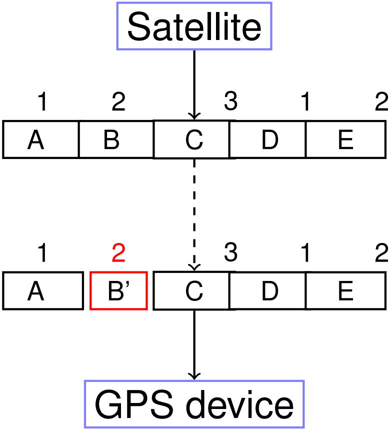

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Agreements per point	: 2.	1 collision per hole.	

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Collisions	: $\geq n$.	$\geq P-H$ Collisions.	
Agreements per point	: 2.	1 collision per hole.	
Points $Q(x)$ and $P(x)$ agree	: $\geq n$.		

Properties: proof.

$P(x)$: degree $n-1$ polynomial.
Send $P(1), \ldots, P(n+2k)$
Receive $R(1), \ldots, R(n+2k)$
At most k i's where $P(i) \neq R(i)$.

Properties:

(1) $P(i) = R(i)$ for at least $n+k$ points i,
(2) $P(x)$ is unique degree $n-1$ polynomial
that contains $\geq n+k$ received points.

Proof: (1) Sure. Only k corruptions.
(2) Degree $n-1$ polynomial $Q(x)$ consistent with $n+k$ points.
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$\Longrightarrow Q(i) = P(i)$ at n points.

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Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

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For each subset of $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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Assume point 1 is wrong and solve..

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Assume point 2 is wrong

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Runtime: $\binom{n+2k}{k}$ possibilitities.

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Something like $(n/k)^k$ ...Exponential in k!.

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How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
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Where oh where have my little packets gone ...

Ditty...

Oh where, Oh where
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Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Where oh where have my little packets gone ...bad.

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my bad packets be?

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(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
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Where oh where can my bad packets be?

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}
(1-e)(p_2+p_1+p_0) &\equiv (3)(1-e) \pmod 7 \\
(2-e)(4p_2+2p_1+p_0) &\equiv (1)(2-e) \pmod 7 \\
(3-e)(2p_2+3p_1+p_0) &\equiv (3)(3-e) \pmod 7 \\
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\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

Example.

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Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}
(1-e)(p_2 + p_1 + p_0) &\equiv (3)(1-e) \pmod{7} \\
(2-e)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-e) \pmod{7} \\
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\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

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4 unknowns (p_0, p_1, p_2 and e),

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\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial! Do know form: $(x - e)$.

4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
\end{aligned}
$$

..turn their heads each day,

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
&\vdots \\
E(i)(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i)E(i) \pmod{p} \\
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E(m)(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m)E(m) \pmod{p}
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...so satisfied, I'm on my way.

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$m = n + 2k$ satisfied equations,

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$m = n+2k$ satisfied equations, $n+k$ unknowns.

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Let $Q(x) = E(x)P(x) = a_{n+k-1}x^{n+k-1} + \cdots a_0$.

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E(m)(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m)E(m) \pmod{p}
\end{aligned}
$$

...so satisfied, I'm on my way.

$m = n + 2k$ satisfied equations, $n + k$ unknowns. But nonlinear!

Let $Q(x) = E(x)P(x) = a_{n+k-1}x^{n+k-1} + \cdots a_0$.

Equations:

$$Q(i) = R(i)E(i).$$

and linear in a_i and coefficients of $E(x)$!

Finding $Q(x)$ and $E(x)$?

Finding $Q(x)$ and $E(x)$?

- $E(x)$ has degree k

Finding $Q(x)$ and $E(x)$?

- $E(x)$ has degree k ...

$$E(x) = x^k + b_{k-1}x^{k-1} \cdots b_0.$$

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$\implies$ k (unknown) coefficients.

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$\implies$ k (unknown) coefficients. Leading coefficient is 1.

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- $Q(x) = P(x)E(x)$ has degree $n + k - 1$

Finding $Q(x)$ and $E(x)$?

- $E(x)$ has degree k ...

$$E(x) = x^k + b_{k-1}x^{k-1} \cdots b_0.$$

$\implies$ k (unknown) coefficients. Leading coefficient is 1.

- $Q(x) = P(x)E(x)$ has degree $n+k-1$ ...

$$Q(x) = a_{n+k-1}x^{n+k-1} + a_{n+k-2}x^{n+k-2} + \cdots a_0$$

Finding $Q(x)$ and $E(x)$?

- $E(x)$ has degree k ...

$$E(x) = x^k + b_{k-1}x^{k-1} \cdots b_0.$$

$\Longrightarrow$ k (unknown) coefficients. Leading coefficient is 1.

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Total unknown coefficient:

Finding $Q(x)$ and $E(x)$?

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$\implies$ k (unknown) coefficients. Leading coefficient is 1.

- $Q(x) = P(x)E(x)$ has degree $n+k-1$ ...

$$Q(x) = a_{n+k-1}x^{n+k-1} + a_{n+k-2}x^{n+k-2} + \cdots a_0$$

$\implies$ $n+k$ (unknown) coefficients.

Total unknown coefficient: $n+2k$.

Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k$,

$$Q(i) = R(i)E(i) \pmod{p}$$

Solving for $Q(x)$ and $E(x)$...

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$$a_{n+k-1} + \ldots a_0 \quad \equiv \quad R(1)(1 + b_{k-1} \cdots b_0) \pmod{p}$$

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$$\begin{aligned} a_{n+k-1} + \ldots a_0 &\equiv R(1)(1 + b_{k-1} \cdots b_0) \pmod{p} \\ a_{n+k-1}(2)^{n+k-1} + \ldots a_0 &\equiv R(2)((2)^k + b_{k-1}(2)^{k-1} \cdots b_0) \pmod{p} \\ &\vdots \end{aligned}$$

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&\vdots \\
a_{n+k-1}(m)^{n+k-1} + \ldots a_0 &\equiv R(m)((m)^k + b_{k-1}(m)^{k-1} \cdots b_0) \pmod{p}
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..and $n+2k$ unknown coefficients of $Q(x)$ and $E(x)$!

Solving for $Q(x)$ and $E(x)$...

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Solve for coefficients of $Q(x)$ and $E(x)$.

Solving for $Q(x)$ and $E(x)$...and $P(x)$

For all points $1,\ldots,i,n+2k$,

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..and $n+2k$ unknown coefficients of $Q(x)$ and $E(x)$!

Solve for coefficients of $Q(x)$ and $E(x)$.

$$\text{Find } P(x) = Q(x)/E(x).$$

Solving for $Q(x)$ and $E(x)$...and $P(x)$

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Solve for coefficients of $Q(x)$ and $E(x)$.

Find $P(x) = Q(x)/E(x)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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$$a_3 + a_2 + a_1 + a_0 \equiv 3(1 - b_0) \pmod 7$$

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$$
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a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\
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a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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\end{aligned}$$

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$Q(x) = x^3 + 6x^2 + 6x + 5$.

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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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Example: finishing up.

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```
          ------------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2
           -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                1 x^2
         -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                1 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x
        -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
              1 x^2 + 1 x + 1
       -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
              1 x^2 + 6 x + 5
              1 x^2 - 2 x
              ---------------
                        x + 5
                        x - 2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
        -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
               1 x^2 + 6 x + 5
               1 x^2 - 2 x
               ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
         -----------------
     x - 2 ) x^3  + 6 x^2 + 6 x + 5
             x^3  - 2 x^2
             ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
            -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
               1 x^2 + 6 x + 5
               1 x^2 - 2 x
               ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1   Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                   1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1   Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

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Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

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Proof:
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$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Berlekamp-Welsh algorithm decodes correctly when k errors!

Summary. Error Correction.

Communicate n packets, with k erasures.

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Reed-Solomon codes.

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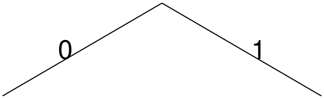
Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..



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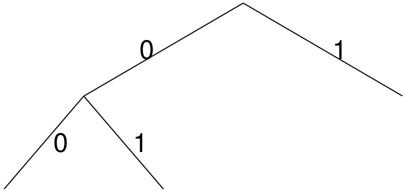

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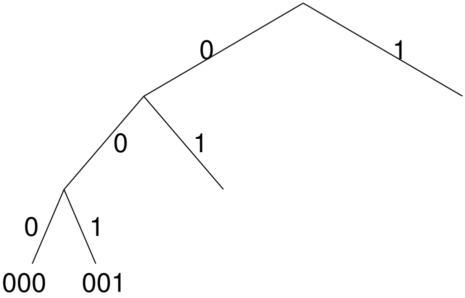

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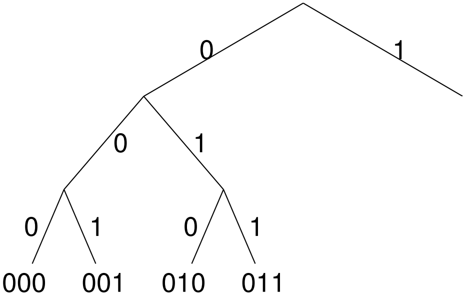

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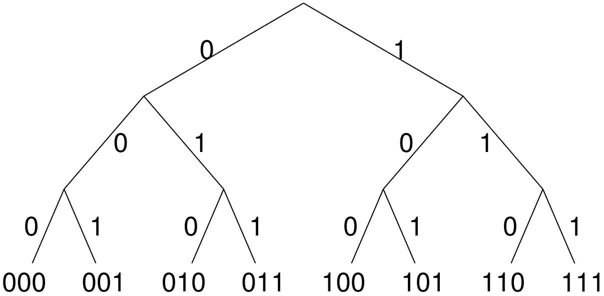

8 leaves which is $2 \times 2 \times 2$.

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, $\ldots$, then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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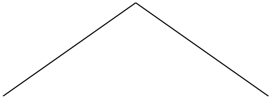

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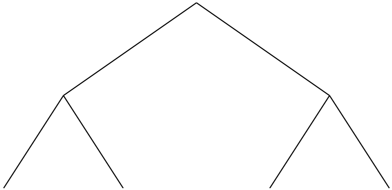

n_1

$\times n_2$

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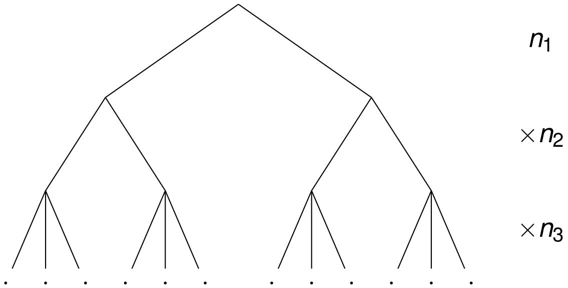

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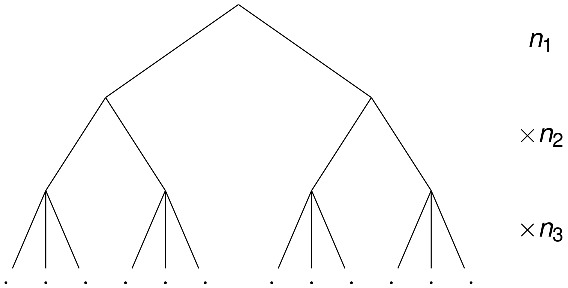

In picture, $2 \times 2 \times 3 = 12$!

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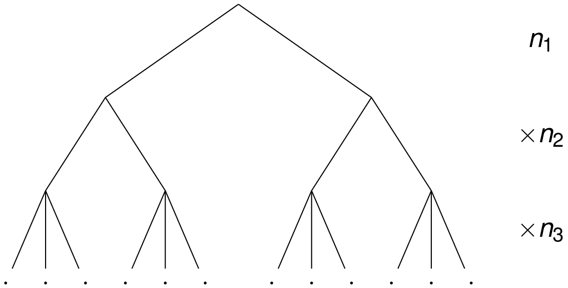

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Using the first rule..

How many outcomes possible for k coin tosses?

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How many outcomes possible for k coin tosses?

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$2 \times 2 \cdots 2 = 2^k$

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How many functions f mapping S to T?

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Counting:Summary.

Today: How many permutations of “CAT”?

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....3

Counting:Summary.

Today: How many permutations of "CAT"?

....3 $\times$

Counting:Summary.

Today: How many permutations of “CAT”?

....3×2

Counting:Summary.

Today: How many permutations of “CAT”?

....$3 \times 2 \times 1$.

Counting:Summary.

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....$3 \times 2 \times 1$.

How many permutations of “ANAGRAM”?

Counting:Summary.

Today: How many permutations of “CAT”?

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How many permutations of “ANAGRAM”?

Wednesday!