CS70: Jean Walrand: Lecture 27.

CLT + Random Thoughts

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1. Review: Continuous Probability
2. Normal Distribution
3. Central Limit Theorem
4. Random Thoughts

Continuous Probability

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7. Gaussian: $\mathcal{N}(\mu, \sigma^2) : f_X(x) = ...$ “bell curve”

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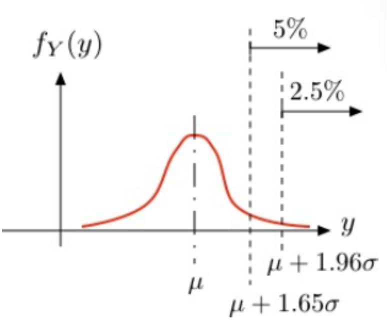

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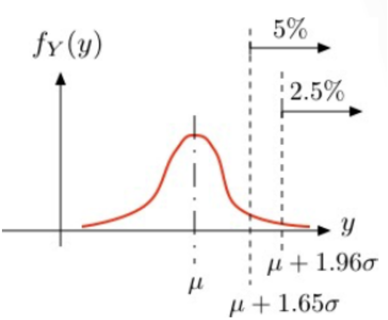

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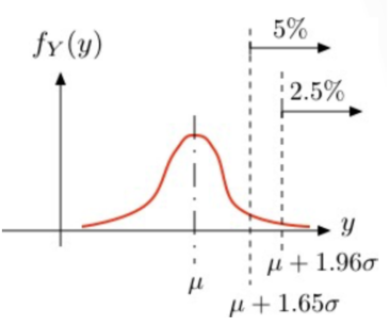

Note: $Pr[|Y-\mu| > 1.65\sigma] = 10\%$; $Pr[|Y-\mu| > 2\sigma] = 5\%$.

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Expectation, Variance.

Theorem If $Y = \mathcal{N}(\mu, \sigma^2)$, then

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Central limit theorem: As n goes to infinity the distribution of S_n approaches the standard normal distribution.

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That is,

$$Pr[S_n \leq \alpha] \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-x^2/2} dx.$$

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Proof: See EE126.

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Thus, for $n \gg 1$, one has

$$Pr[-2 \leq |\frac{A_n - \mu}{\sigma/\sqrt{n}}| \leq 2] \approx 95\%.$$

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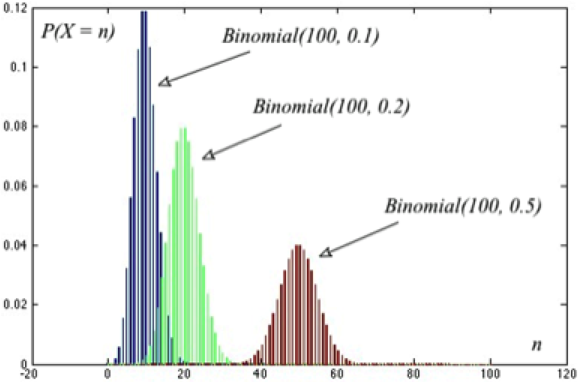

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Female students happen to apply more to the college that admits fewer students.

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- Beware of statistics reported in the media!

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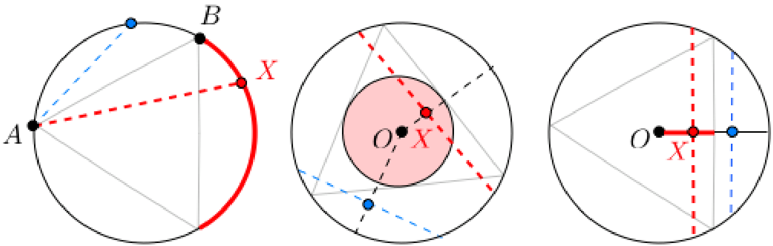

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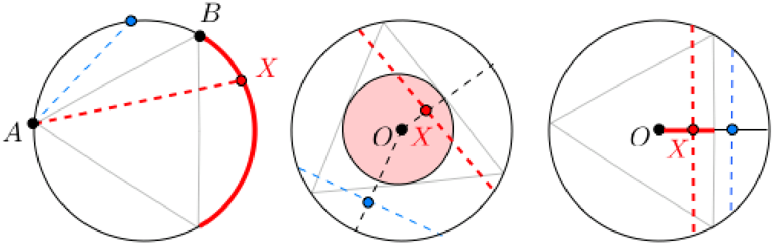

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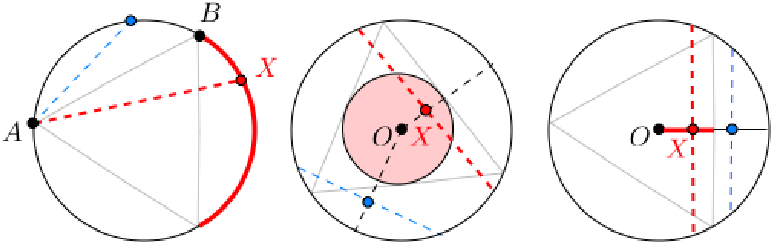

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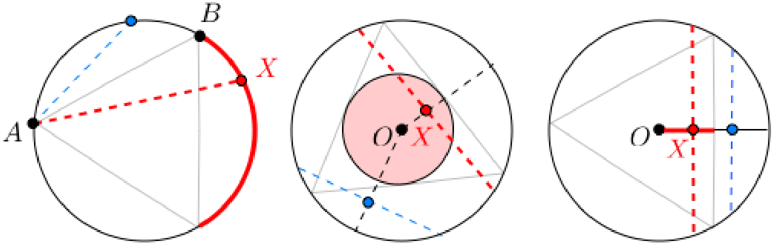

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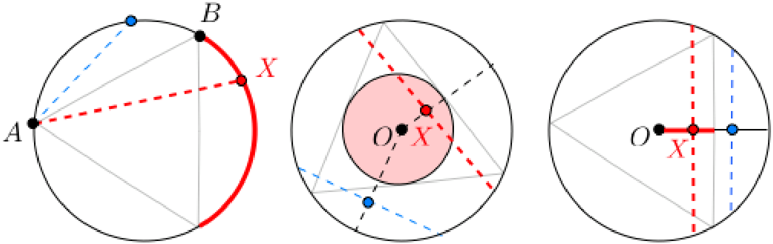

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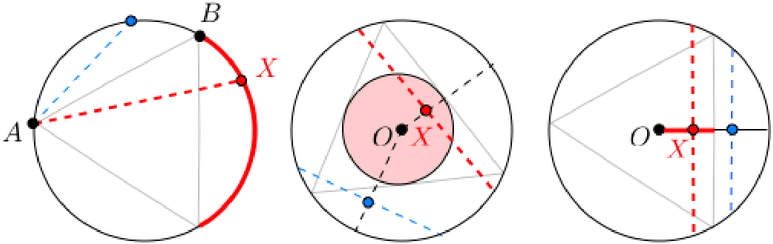

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As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

Confirmation Bias: An experiment

There are two bags. One with 60% red balls and 40% blue balls; the other with the opposite fractions.

One selects one of the two bags.

As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

Surprisingly, people tend to be reinforced in their original belief, even when the evidence accumulates against it.

Being Rational: ‘Thinking, Fast and Slow’

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In this book, Daniel Kahneman discusses examples of our irrationality.

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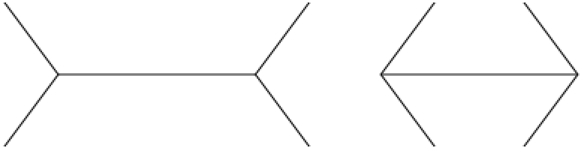
- Perception illusions: Which horizontal line is longer?



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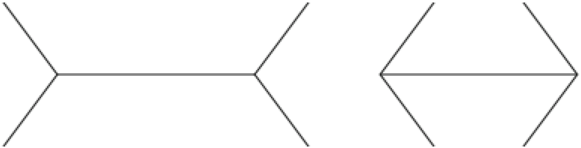

It is difficult to think clearly!

What to Remember?

Professor,

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Professor, what should I remember about probability from this course?

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Key Ideas in CS70 Probability

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- Descriptive:

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- Descriptive: $Pr[A], E[X],$

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Key Ideas in CS70 Probability

- Descriptive: $Pr[A], E[X], E[h(X,Y)], \pi_n, \beta(i), \alpha(i)$
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- Prescriptive: How to play a game, how to design, ...

What's Next?

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Professors, I loved this course so much!

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Professors, I loved this course so much! I want to learn more about discrete math and probability!

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Final Thoughts

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More precisely:

Final Thoughts

More precisely: Some thoughts about the final ....

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How to study for the final?

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How to study for the final?

- Lecture Slides;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours,

Final Thoughts

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How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours,

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- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours, Reviews by TAs

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- Lecture Slides; Notes; Discussion Problems; HW
- TA Office Hours, Prof. Office Hours, Reviews by TAs
- Next week: reviews during normal lecture hours:

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More precisely: Some thoughts about the final ....

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  - Discrete Math (Tuesday);

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More precisely: Some thoughts about the final ....

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- TA Office Hours, Prof. Office Hours, Reviews by TAs
- Next week: reviews during normal lecture hours:
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  - Probability (Thursday).

Parting Thoughts

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You have learned a lot in this course!

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Proofs,

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Proofs, Graphs,

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Proofs, Graphs, Mod(p),

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Proofs, Graphs, Mod(p), RSA,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... , how to handle stress,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... , how to handle stress, how to sleep less,

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See you on Tuesday.