1   Count It!

For each of the following collections, determine and briefly explain whether it is finite, countably infinite (like the natural numbers), or uncountably infinite (like the reals):

(a) $\mathbb{N}$, the set of all natural numbers.

(b) $\mathbb{Z}$, the set of all integers.

(c) $\mathbb{Q}$, the set of all rational numbers.

(d) $\mathbb{R}$, the set of all real numbers.

(e) The integers which divide 8.

(f) The integers which 8 divides.

(g) The functions from $\mathbb{N}$ to $\mathbb{N}$.

(h) Computer programs that halt.

(i) Computer programs that always correctly tell if a program halts or not.

(j) Numbers that are the roots of nonzero polynomials with integer coefficients.

2   Halting Problem Sanity Check

Suppose you want to prove that a program A is uncomputable. Which of the following should you do?

(a) Show that A can be solved if the halting problem could be solved.

(b) Show that the halting problem could be solved if A could be solved.

3 Code Reachability

Consider triplets (M, x, L) where

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M is a Java program
x is some input
L is an integer
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and the question of: if we execute $M(x)$, do we ever hit line L?

Prove this problem is undecidable.

4 Hello World!

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program P on input x outputs "Hello World!". Is there a computer program that can perform this task? Justify your answer.

(b) You want to determine whether a program P prints "Hello World!" before running the kth line in the program. Is there a computer program that can perform this task? Justify your answer.

(c) You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task? Justify your answer.