Section 6

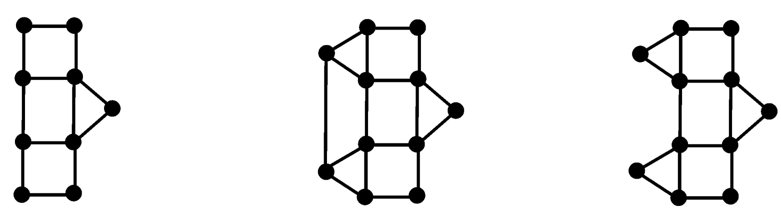
Monday, July 15

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. Tom wants to share a secret among his 4 TAs and 14 readers, such that a subset of them can reconstruct the secret iff it contains either (i) at least 2 TAs, or (ii) at least 1 TA and at least 2 readers, or (iii) at least 4 readers. Explain how this can be accomplished. How large does the finite field need to be?

2. Suppose Alice wants to transmit to Bob a polynomial P of degree ≤ 1 over $GF(5)$. She sends packets indicating the values of $P(0)$, $P(1)$, $P(2)$, and $P(3)$ so that Bob will be able to recover P even if two packets are dropped. However, a disaster happens and three packets are dropped: Bob only receives a packet indicating that $P(2) = 3$. Help Bob find a list of all the polynomials that P could have been, given this information. (Note that Bob already knows that the polynomial has degree ≤ 1 and is over $GF(5)$.)

3. For each of the following three graphs, determine whether you can draw all the edges without picking up your pencil and without retracing any edge (you do not need to end up where you started). In other words, find an eulerian path or prove that none exists!



4. Here are some questions about eulerian and hamiltonian cycles.

   4a. Recall that an eulerian cycle uses every edge in a graph exactly once. Some connected graphs have eulerian cycles and some do not. In contrast, prove that *every* connected undirected graph has a cycle that uses every edge exactly *twice*. (Usually, the definition of a cycle does not allow an edge to be used more than once, but here we obviously must allow it.)

   4b. Give an example of an undirected graph G with no isolated vertices, such that G has a hamiltonian cycle but does not have an eulerian cycle.

   4c. Give an example of an undirected graph G with no isolated vertices, such that G has an eulerian cycle but does not have a hamiltonian cycle.

5. You have two containers, one with capacity a liters and the other with capacity b liters, where a and b are positive integers. Initially they are empty, and you want the capacity-a container to have c liters of water, and the capacity-b container to have d liters of water, where c and d are nonnegative integers. The only things you can do are (1) empty a container, (2) fill a container to the top (you have unlimited water), and (3) pour one into the other until either the container you're pouring into becomes full or the container you're pouring out of becomes empty, whichever comes first. (A classic version of this puzzle has $a = 3$, $b = 5$, $c = 0$, and $d = 4$.) Explain how to model this as a graph problem.