1. Visualizing Error Correction

Alice wants to send a message of 2 packets to Bob, and wants to guard against 1 lost packet. So working over $GF(3)$, she finds the unique polynomial $P(x)$ that passes through the points she wants to send, and sends Bob her augmented message of 3 packets: $(0, P(0)), (1, P(1)), (2, P(2))$.

One packet is lost, so Bob receives the following packets: $(0, 2), (2, 0)$.

(a) Plot the points represented by the packets Bob received on the grid below.

(b) Draw in the unique polynomial $P(x)$ that connects these two points.

(c) By visual inspection, find the lost packet $(1, P(1))$.

2. Erasure Warm-Up

Working over $GF(q)$, you want to send your friend a message of $n = 4$ packets and guard against 2 lost packets. What is the minimum $q$ you can use? What is the maximum degree of the unique polynomial that describes your message?

3. Aliens, Oh My!

Alice wants to send a plea for help to an alien space ship that is hovering near her city. She knows that at their current distance of 7 miles above ground, no more than 3 general errors can occur during transmission. If she sends a message of length 15, how long must her original message be?
4. Where Are My Packets?

Alice wants to send the message \((c_0, c_1, c_2)\) to Bob, where each \(c_i \in \{0, 1, 2, 3, 4\}\). She encodes it as a polynomial \(P\) of degree \(\leq 2\) over \(GF(5)\) such that \(P(0) = c_0\), \(P(1) = c_1\), and \(P(2) = c_2\), and she sends the packets \((0, P(0)), (1, P(1)), (2, P(2)), (3, P(3)), (4, P(4))\). Two packets are dropped, and Bob only learns that \(P(0) = 4\), \(P(3) = 1\), and \(P(4) = 2\). Help Bob recover Alice’s message.

(a) Find the multiplicative inverses of 1, 2, 3 and 4 modulo 5.
(b) Find the original polynomial \(P\) by using Lagrange interpolation or by solving a system of linear equations.
(c) Recover Alice’s original message.

5. More Erasures!

Consider the alphabet \(A = 0\), \(B = 1\), \(C = 2\), \(D = 3\), \(E = 4\). Suppose a message of length 3 is sent using the error correction scheme discussed in class over \(GF(5)\). If you receive the following packets, what was the original message?

(a) \(C \_ A A\)
(b) \(\_ A C C\)
(c) Can you determine the original message if you only receive \(C E \_ \_\)? Either find the original message or explain why you can’t.