CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 5A

1. Boy or Girl?

The following are variants of the famous "boy or girl paradox".

Note: For both parts, assume that the probability of a boy or girl being born is the same, a child is equally likely to be born on any day of the week, and the genders of all children are independent of each other and independent of the day of the week.

(a) Mr. and Mrs. Smith have two children, at least one of whom is a girl. What is the probability that both children are girls?

(b) Mr. and Mrs. Brown have two children, one of whom is a boy born on a Tuesday. What is the probability that both children are boys?

2. Wedding in the Desert

Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year, so the prior probability of rain is just $5/365$. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 5% of the time. What is the probability that it will rain on the day of Marie's wedding?

3. Three Diseases

A doctor assumes that a patient has exactly one of three diseases d_1, d_2, or d_3. Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has d_1, 0.6 if the patient has disease d_2, and 0.4 if the patient has disease d_3. Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

4. Dice Probability

We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

(a) Find the probability that doubles were rolled.

(b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

(c) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.