EE-100 Lab: Op Amp Gyrator Circuit Synthesis and Applications - Theory

1. Objective

In this laboratory measurement you will learn about the *gyrator*, its op amp circuit synthesis, and some possible gyrator applications.

2. Introduction: Gyrator

A gyrator is an ideal two-port element defined by the following equations:

$$i_1 = G \cdot v_2$$

$$i_2 = -G \cdot v_1$$

where the constant G is called the *gyration conductance*. The symbol for a gyrator is shown in Figure 1.



Figure 1 An ideal gyrator

Properties of the ideal gyrator: It is easy to check that the ideal gyrator is a *non-energic* element, i.e., at all times the power delivered to the two-port is identically zero. *Proof:* the total instantaneous power entering a gyrator is

$$p(t) = v_1 \cdot i_1 + v_2 \cdot i_2 = v_1 \cdot (G \cdot v_2) + v_2 \cdot (-G \cdot v_1) = 0$$

The fundamental property of an ideal gyrator is given by the following equation:

$$\frac{v_1}{i_1} = \frac{-i_2/G}{G \cdot v_2} = \frac{1}{G^2} \cdot \frac{-i_2}{v_2}$$

That is, when a gyrator is terminated at the output port with an $R_L \Omega$ linear resistor as shown in Figure 2, the input port behaves as a linear resistor with resistance $1/(G^2R_L)$.



Figure 2 A gyrator terminated at the output port with a resistor.

Capacitor-to-Inductor Mutation Property: An interesting property is the following: if the output port of an ideal gyrator is terminated with a capacitor as shown in Figure 3, the input port behaves like an inductor. Thus a gyrator is a useful element in the design of inductorless filters. *Proof:* the input voltage of the gyrator is

$$v_1 = -\frac{1}{G} \cdot i_2 = -\frac{1}{G} \cdot \left(-C\frac{dv_2}{dt}\right) = \frac{C}{G} \cdot \left(\frac{1}{G} \cdot \frac{di_1}{dt}\right) = \underbrace{\left(\frac{C}{G^2}\right)}_L \cdot \left(\frac{di_1}{dt}\right) = L \cdot \frac{di_1}{dt}$$

where L inductance is C/G^2 .



Figure 3 A gyrator terminated at the output port with a capacitor behaves like an inductor.

Current Source to Voltage Source Mutation Property: If the output port of an ideal gyrator is terminated with a voltage source as shown in Figure 4, the input ports behaves like a current source. Similarly, connecting a current source across the output port of a gyrator we get a voltage source.



Figure 4 A gyrator terminated at the output port with a voltage source behaves like a current source.

Driving Point Characteristic Reflection Property: Another interesting observation is the following. If the output port of a gyrator is connected across a current-controlled two-terminal resistor, i.e., $v_2=f(-i_2)$, then the input port becomes a voltage-controlled resistor as shown in Figure 5. The resulting voltage-controlled resistor is then the dual of the original current-controlled resistor. Similarly, if we connect a voltage-controlled resistor we get its dual current-controlled resistor.



Figure 5 A gyrator terminated at the output port with a current controlled resistor behaves like a voltage controlled resistor.

3. Op Amp Gyrator Synthesis

Now, we will present some ways for making a gyrator using components which are available commercially. We will focus on op amp realization only. *Remark: Physical gyrators which approximate the property of an ideal gyrator over low operating frequencies (below 10 kHz) are available commercially in the form of integrated circuit modules.*

Note that the basic equation of an ideal gyrator contains port currents controlled by port voltages. Using VCCS (voltage-controlled current source) as building blocks (shown in Figure 6) we can realize a gyrator by connecting them in parallel, as shown in Figure 7.



Figure 6 (a) VCCS – voltage-controlled current source

(b) VCCS realization with op amp



Figure 7 Gyrator realized by connecting two VCCS-s in parallel.

The drawback of this gyrator realization is that the two op amp circuits cannot work from the same dual power supply they would require two independent (floating) dual power supplies.

A single dual power supply implementation is shown in Figure 8. This will be used in your laboratory measurement. A good exercise is to derive the port equations.



Figure 8 Gyrator realization using two op amps with single dual power supply.

Next we show two circuits realizing a current source and an inductor both via gyrator. These test circuits will be measured in the laboratory experiment.



Figure 9 Current source realization via Gyrator

*E*₀: 1, 2, 3V and -1, -2 -3V. *R*_L: 100Ω, 430Ω, 1k, 2k



Figure 10 Inductor realization via Gyrator

C: 10nF, 220nF, or 1uF *R*_{*L*}: 1k

EE-100 Lab: Op Amp Gyrator Circuit Synthesis and Applications – Experiment Guide

Exp1 – Current source realization via Gyrator

Build the circuit shown in Figure 9. ($V_{DD} = 12V$ and $V_{SS} = -12V$). Let $E_0 = 1V$, 2V, 3V and $R_L = 100 \ \Omega$. You will be provided a special box designed for this laboratory measurement (Figure 11). Measure the output current (i_2) and the output voltages of the operational amplifier (v_{01} and v_{02}). Repeat the measurement with different load resistor values, e.g. $R_L = 430$, 1k, 2k and different voltage sources $E_0 = -1V$, -2V, -3V. Derive and verify the conditions necessary for the op amp implementation to work as a current source. What is the maximum value of the load resistor for this current source realization if it is designed to work over the current range [$-3 \ mA$; $3 \ mA$]?

(Hint: The circuit works as a current source only in the linear operating region of the op amp. Output voltage of the op amp cannot exceed its saturation voltage level, appr. V_{DD} or V_{SS}).

Exp2 – Inductor realization via Gyrator

Build the circuit as shown in Figure 10. ($V_{DD} = 12V$ and $V_{SS} = -12V$). Let C be 10nF, 220nF, or 1uF. (You need to measure only one case). Let R_L be 1k.Verify that the circuit works as an inductor (i.e., whether the input impedance is inductive, in the sense that the current lags the voltage by 90⁰). Set the output of the function generator to a $1V_{pp}$, 1kHz sine wave with 0 DC offset. Using the scope, display and measure both the voltage and the current of our "*inductor*". Check whether the current of the "inductor" lags its voltage. You can use the X/Y mode of the scope as well. (*Hint: the "inductor" current - i*₂ is related to the voltage across the load resistor (R_L). You can measure this current and the voltage of the function generator only if the function generator is not a floating type device – see important note below^{*}. If this is the case, then you cannot observe directly a 90⁰ phase shift but rather the joint behavior of an RL circuit.)

(b) - Set different sine wave frequencies (from 100 Hz, to 10kHz), and repeat the previous measurement. What is the frequency when there is an exact 90^0 phase shift? Explain the reason of this behavior.

*IMPORTANT NOTE ABOUT EARTH GROUND: The following caution applies to both the HP33120A function generator and the oscilloscope. The building earth ground is connected to the ground wire of the instruments (outside ring of each of the BNC probe connectors is connected to this ground). If you put the ground clips of probe A1 and probe A2 of the oscilloscope at two different nodes, the ground will short both nodes together. If you put the ground of the function generator at a different node then this will also be connected to the earth ground. So always take care on how you set up your measurement and avoid connecting nodes together that are considered to be separate. **Appendix**: You will use a special box designed for this laboratory measurement (Figure 11).



Figure 11 Box designed for gyrator laboratory measurement

The arrangement of the components of box matches the corresponding schematic. The box requires power supply refer to the basic lab guide what the proper connection is. You can see labels showing which banana socket belongs to the positive power supply (V+= +12V, and red color), the common terminal (always in the middle), and the negative power supply (V- = -12V). Do not mix them. If you fill unsure ask your TA to help you. Passive components (resistors, capacitors, etc) are prepared in advance you need only to put them to the proper small banana sockets. Large banana sockets on the left side of the box are used for input, while sockets on the right for the output. If you need to set DC input voltage you can use the 6V output of the power supply. To set periodic input voltage using the function generator you can use BNC-banana converters. Also, there are prepared banana plugs with small pins so you can connect the alligator clips of the scope probe and measure any desired signal.