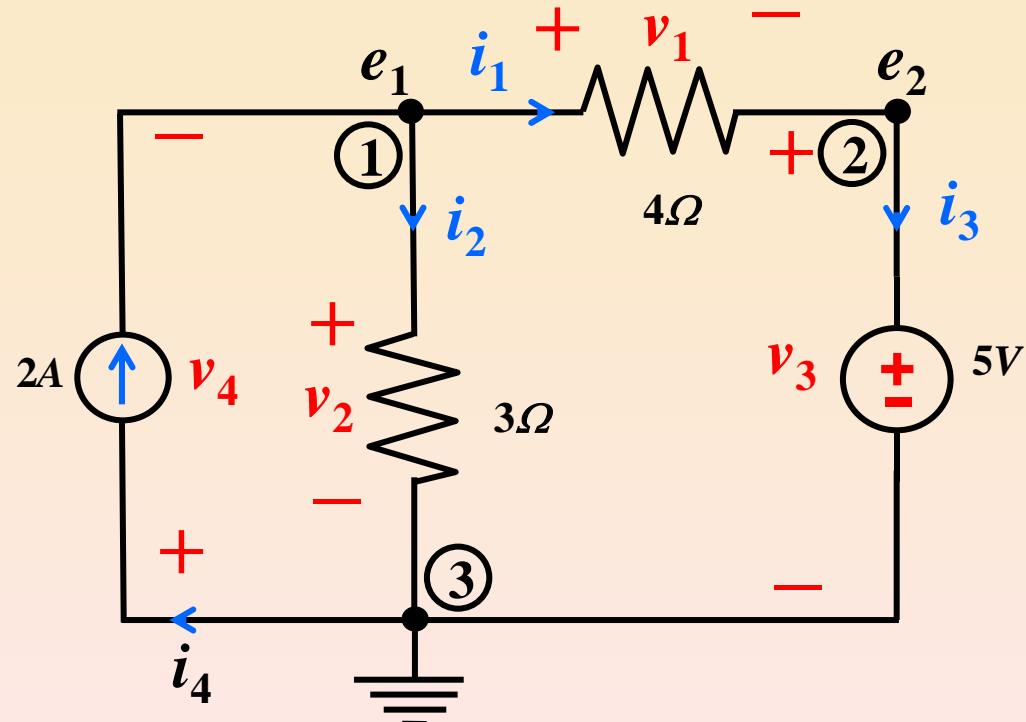
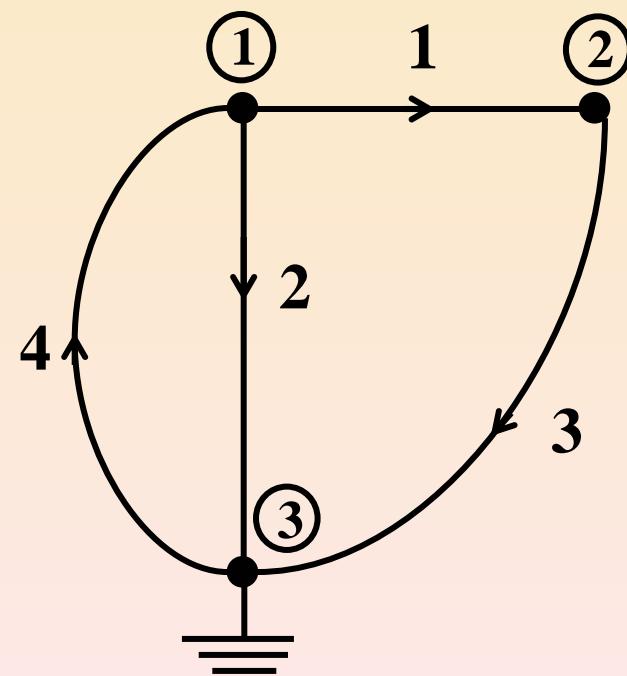


Circuit N



Digraph G



Reduced Incidence Matrix A

node number	branch number			
①	1	2	3	4
②	[1 1 0 -1]			
③	[-1 0 1 0]			

Let \mathcal{N} be **any** connected circuit made of arbitrary collection of 2-terminal, 3-terminal, and m -terminal devices. Let the **digraph** G associated with \mathcal{N} contain “ n ” nodes and “ b ” branches.

Denote

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix}, \quad \dot{\mathbf{i}} = \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \vdots \\ \dot{i}_b \end{bmatrix}$$

e_i
 v_j
 \dot{i}_k

\mathbf{e}
 \mathbf{v}
 $\dot{\mathbf{i}}$

e_i
 v_j
 \dot{i}_k

e_i
node-to-datum
voltage vector

v_j
branch
voltage vector

\dot{i}_k
branch
current vector

There are all together $2b + (n-1)$ **circuit variables** to be determined for each circuit \mathcal{N} .

Reduced Incidence Matrix A

Let G be a connected **digraph** with “ n ” **nodes** and “ b ” **branches**. Pick any node as the **datum node** and label the remaining nodes arbitrarily from ① to ⑩. Label the branches arbitrarily from 1 to b .

The **reduced incidence matrix A** of G is an $(n-1) \times b$ matrix where each **row** j corresponds to **node** j , and each **column** k , corresponds to **branch** k , and where the jk th element a_{jk} of **A** is constructed as follow:

$$a_{jk} = \begin{cases} 1 & , \text{ if branch } k \text{ leaves node } j \\ -1 & , \text{ if branch } k \text{ enters node } j \\ 0 & , \text{ if branch } k \text{ in not connected to node } j \end{cases}$$

How to write An Independent System of KCL and KVL Equations

Let N be any connected circuit and let the **digraph** G associated with N contain “ n ” nodes and “ b ” branches. Choose an arbitrary datum node and define the associated **node-to-datum voltage** vector \mathbf{e} , the **branch voltage vector** \mathbf{V} , and the **branch current vector** \mathbf{i} . Then we have the following system of **independent** KCL and KVL equations.

(n-1) Independent KCL Equations :

$$\mathbf{A} \mathbf{i} = \mathbf{0}$$

b Independent KVL Equations :

$$\mathbf{V} = \mathbf{A}^T \mathbf{e}$$

KCL:

$$\underbrace{\begin{bmatrix} 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}}_{\mathbf{i}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{0}} \Rightarrow \boxed{\begin{aligned} i_1 + i_2 - i_4 &= 0 \\ -i_1 + i_3 &= 0 \end{aligned}}$$

KVL:

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}_{\mathbf{V}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}}_{\mathbf{A}^T} \underbrace{\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}}_{\mathbf{e}} \Rightarrow \boxed{\begin{aligned} v_1 &= e_1 - e_2 \\ v_2 &= e_1 \\ v_3 &= e_2 \\ v_4 &= -e_1 \end{aligned}}$$