The choice of the term "black box" is quite appropriate here because the box is really black inside in the sense that we cannot see its contents. As a matter of fact, unless we open the box and peep inside, there is no way of determining its contents. However, as engineers, we are not so much interested in the contents of the box as in knowing what the black box can do and how it behaves externally when it is connected with other black boxes into a network. In other words, we are primarily interested in predicting the external behavior of the black box without having to perform any tedious experiment. Our first step toward such an analytical approach is to "characterize" the black box. The concepts involved in characterizing a black box are so important that we pause here to consider a simple but illustrative analogy.

1-5-1 A MECHANICAL BLACK-BOX ANALOGY

Suppose we are given the mechanical black box containing a "spring" as shown in Fig. 1-7*a*. Suppose we did not know the contents of this black box and were asked to predict the behavior of the external terminals when an arbitrary force $f(t)$ is applied to terminal a of the black box with terminal b fixed against the wall. The mechanical variables of interest here are the displacement x (displacement to the right of the initial position 0 is assumed positive), the velocity v (of terminal a), and the force f (positive for tension and negative for compression). Clearly, the only way we can hope to characterize this black box (other than opening the box) is to start performing some experiments. Suppose we begin by applying a constant force $f = A$ and measure the corresponding velocity of terminal a. This would give us a point in the velocity-vs.-force plane (f-v plane). By repeating the above experiment with several values of force f, we obtain the data shown in Fig. 1-7*b*. We might be tempted to draw a smooth curve through

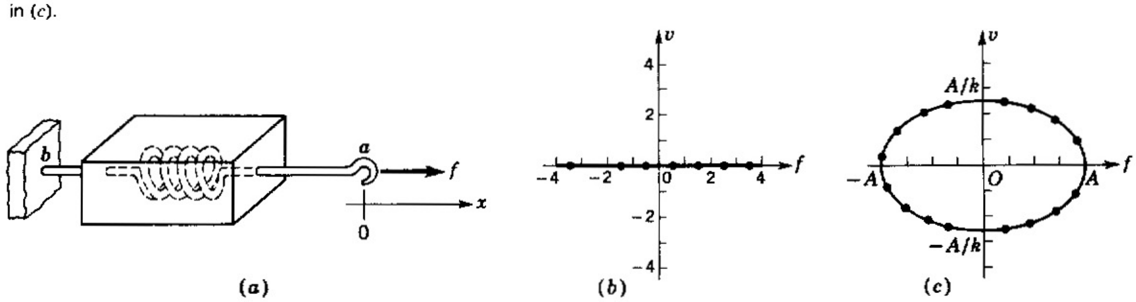


Fig. 1-7. An example illustrating the characterization of a mechanical black box. The data points in the v-vs.-f plane were found to lie on the horizontal axis in (*b*) and on the ellipse in (*c*).

these data points (which in this case happen to be the f axis) and claim to have characterized the black box in the sense that given any constant force f, we can analytically predict the associated velocity. However, a little thought will show that we have not really characterized the black box yet, for if, instead of applying a constant force we apply a slowly varying sinusoidal force, $f(t) = A \sin t$. The above characteristics would predict that $v(t) = 0$. This is of course contrary to what we expect to observe experimentally; namely, $v(t) = (A/k) \cos t$ where k is the "spring constant." We might hope that this inconsistency can be resolved by plotting all points (f,v) satisfying the above equations and obtaining an ellipse as shown in Fig. 1-7*c*. Observe, however, that the length of both axes of the ellipse depends on the amplitude A of the sinusoidal force, and for each value of A we would obtain a corresponding ellipse, so that eventually the entire f-v plane would be filled up with data points. Moreover, even if we can draw an infinite set of ellipses, we would be able to predict the velocity only if $f(t)$ is sinusoidal. Using these ellipses to predict v due to nonsinusoidal $f(t)$ would again yield erroneous answers. Reluctantly, we must admit that our efforts so far have been in vain and that just about the only useful information we obtained from the above experiment is that the black box cannot be characterized by a curve in the f-v plane.

Let us try another set of variables, say the force f and the displacement x, and repeat the experiments. As before, we begin by applying a constant force $f = A$ and measure the corresponding displacement x. Repeating this for various values of f, we obtain the data points shown in Fig. 1-8*a*. If we draw a smooth curve through these points, we obtain a single relationship

$$x = T(f)$$

Before we try to draw any conclusion, however, our previous experience suggests that we repeat the experiment with time-varying forces to see whether the above relationship still holds. Carrying out the proposed experiment with several low-frequency sinusoidal waveforms as before, we find that at any time $t = t_0$, the data point $[f(t_0),x(t_0)]$ always falls on the same curve $x = T(f)$. This is very encouraging, but to be sure, we must try some other nonsinusoidal waveforms for $f(t)$. Again, we find that, provided $f(t)$ does not change very rapidly,[1] the data point at any time also agrees with the curve in Fig. 1-8*a*. Hence, we can now draw the following conclusion: *For any $f(t)$ which does not change rapidly,*

[1] This condition is actually equivalent to the statement that the frequency of the sinusoidal waveform is not very high. This will become obvious after the reader studies signal analysis.

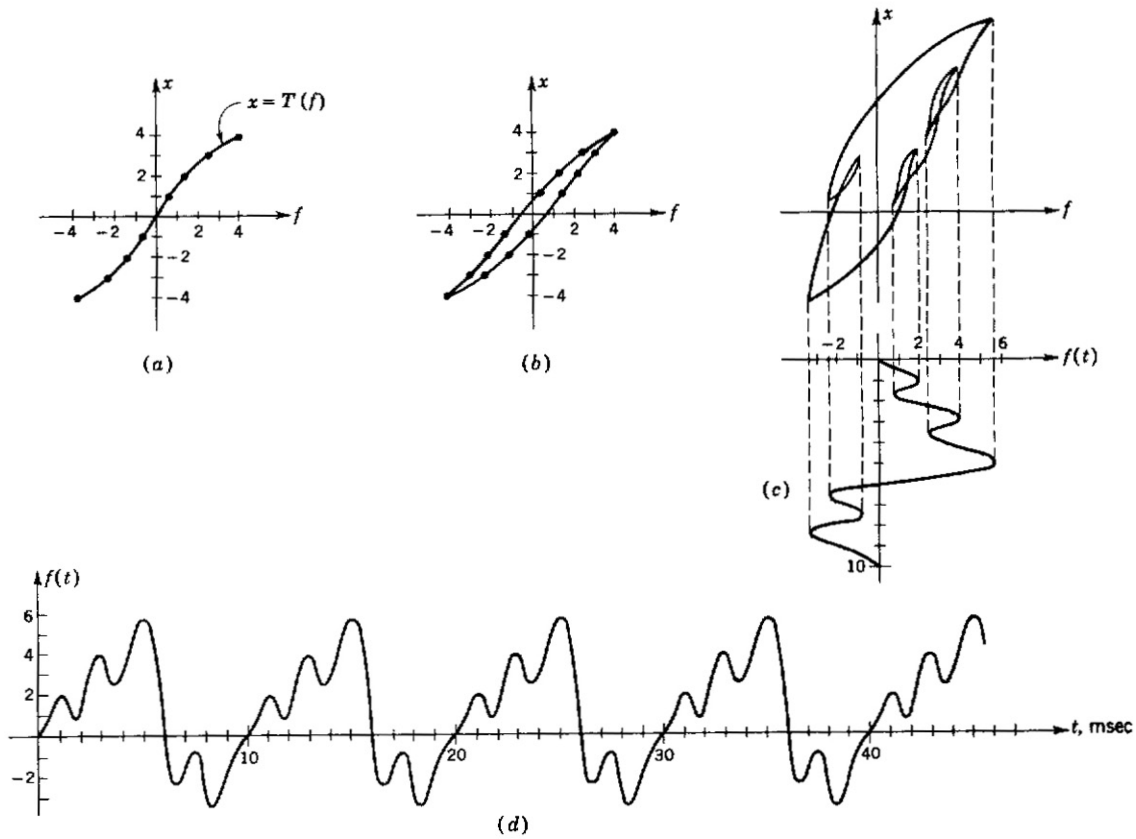


Fig. 1-8. As the frequency of the forcing function $f(t)$ increases, the x-vs.-f characteristic of the mechanical black box changes from a monotonic curve to a hysteresis loop.

the black box can be characterized by the displacement-vs.-force (f-x) curve shown in Fig. 1-8a.

After experiencing the length of time needed to carry out the above experiments, we can now begin to appreciate the utility of such a conclusion; namely, the characterization of the black box permits an analytical solution and thereby eliminates the need to carry out any further experiments.

Observe that our conclusion is based on the assumption that $f(t)$ does not change rapidly. Let us now repeat our experiment with higher-frequency sinusoidal waveforms, as well as with non-sinusoidal waveforms which change rapidly. The experiment shows that as we increase the frequency of the sinusoidal force $f(t)$, the data points begin to deviate (rather slowly at first) from the predicted curve $x = T(f)$. As we increase the frequency further, the data points begin to form a closed loop as shown in Fig. 1-8*b*, and the area enclosed by the loop tends to increase with frequency. Similarly, we find that if we apply a nonsinusoidal force which changes rapidly with time, the deviation from the curve in Fig. 1-8*a* is even worse. For example, Fig. 1-8*c* shows the f-x curve corresponding to the high-frequency nonsinusoidal waveform shown in Fig. 1-8*d*. The above experimental result shows that our earlier assumption, that $f(t)$ should not change very rapidly, is indeed necessary. In order to emphasize this restriction, it is a common practice to call the relationship obtained in Fig. 1-8*a* a *static* characteristic curve in contrast to the *dynamic* characteristic curve which corresponds to measurements at higher frequencies. Since the deviation of the measured characteristic curve from the static characteristic increases slowly with frequency, rather than abruptly, it is impossible to pick a definite frequency above which the static characteristic does not hold. Neither is it possible to find a single dynamic characteristic curve which would hold for all high frequencies. Hence, a certain amount of engineering judgment is involved in deciding whether a certain static characteristic curve can be used satisfactorily to solve a given problem. It is encouraging, however, to know that a large percentage of practical networks can indeed be analyzed by using only static characteristics. Moreover, even in cases when the static characteristic fails to give satisfactory solutions, we shall show in the future that we can often patch up the error by including "parasitic elements," namely, elements which are undesirable but which are invariably present in the black box in minute quantities. For the above example, the parasitic element consists of the *mass* associated with the spring. At low frequencies, the mass, being quite small, has relatively no effect on the measured f-x characteristic. However, as the frequency of the external force $f(t)$ increases, the acceleration of the spring increases, and the inertia force due to the mass becomes appreciable and, in fact, increases as acceleration increases. The deviation of the dynamic characteristic in Fig. 1-8*b* and 1-8*c* from the static characteristic in Fig. 1-8*a* can therefore be attributed to the inertia mass of the spring.

1-5-2 STATIC CHARACTERISTICS OF A TWO-TERMINAL BLACK BOX

The above discussion clearly shows the significance of static characteristics of a black box. Since *all characteristics to be considered in this book are assumed to be static characteristics,* we shall henceforth delete the adjective "static."

Let us now return to the main theme of this section, namely, the characterization of a two-terminal black box. Clearly, the only way we can hope to achieve this is to perform some meaningful external measurements. The only quantities of interest to us are those which can be measured externally. For example, the terminal voltage v and the terminal current i are of primary interest because they can be readily measured. The charge q and the flux linkage φ are also of interest because they can be indirectly measured by *integrating* the measured current waveform $i(t)$ and the measured voltage waveform $v(t)$ in accordance with Eqs. (1-11) and (1-12), respectively. From these measurements, we shall then try to establish a relationship, if there is any, between each pair of *independent* variables. Since the members of the pair of variables i and q are related by Eq. (1-7), they are not independent. Similarly, the variables v and φ are related by Eq. (1-8) and are also not independent. The only remaining combinations consist, therefore, of a relationship between the following variables:

1. Relationship between v and i
2. Relationship between v and q
3. Relationship between i and φ
4. Relationship between q and φ

The last relationship does not occur frequently in practice and has little practical significance. Hence, we shall restrict our attention throughout this book to only the first three cases. These correspond, respectively, to three basic types of two-terminal network elements, namely, a *two-terminal resistor*, a *two-terminal capacitor*, and a *two-terminal inductor*.

Our next step is therefore to plot the data in the v-i, v-q, and i-φ planes, in order to see if the points in any one of these planes can be connected to form a curve. In general, this may not be possible. For example, suppose the two-terminal black box happens to be a capacitance of 1 F. Then from elementary physics, we know that the relationship between v and i is $i = 1(dv/dt)$. But suppose we did not know that the black box contains a capacitance and proceeded to plot the data in the v-i plane. Clearly, it is impossible to expect that a curve can be found which passes through all data points in the v-i plane; in fact, if we take enough measurements, the data points will eventually fill the entire v-i plane. This is easily seen if we apply a voltage source of the form $v(t) = A \sin t$; since $i = dv/dt$, we obtain $i(t) = A \cos t$. Hence at any time $t = t_0$, we obtain a point $(A \sin t_0, A \cos t_0)$ in the v-i plane. Observe next that corresponding to each value of A, the above points form a circle of radius A since $v^2 + i^2 = A^2$. If we vary the value of A from 0 to ∞, we would eventually fill up the v-i plane with data points, and it would be impossible to find a curve passing through these points. On the other hand, if we choose to plot the points in the v-q plane, then these points can be connected by a smooth curve, namely, the line $q = v$. Therefore if a curve can be found which passes through all possible data points in either the v-i, the v-q, or the i-φ plane, then the two-terminal element is completely characterized by that curve.

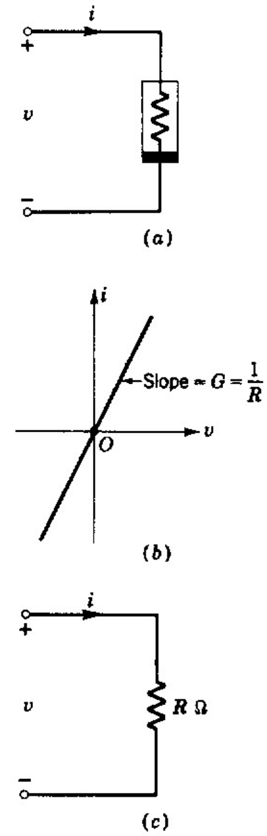


Fig. 1-9. Symbols for a two-terminal resistor.

1-6 TWO-TERMINAL RESISTORS

A two-terminal black box which can be characterized by a curve in the v-i plane is called a *two-terminal resistor* and will be denoted by the symbol shown in Fig. 1-9*a*. Observe that one edge of the symbol is darkened in order to distinguish between the two terminals. This is necessary because the v-i curve measured across the two terminals of a resistor is generally different from that measured across *the same* resistor but with the terminals interchanged (see Prob. 1-1).[1]

1-6-1 LINEAR RESISTORS

Among the infinite variety of v-i curves there is an important subclass which consists of *straight lines passing through the origin* as shown in Fig. 1-9*b*. Resistors of this subclass are called *linear resistors* and will be denoted by the standard symbol shown in Fig. 1-9*c*. Since the v-i curve of a linear resistor is a straight line *through the origin*, it can be described mathematically by $i = Gv$, or $v = Ri$. The constant G represents the slope of the line and is called the *conductance*. The constant R is defined as the reciprocal of G and is called the *resistance*. The practical unit of conductance is the *mho*. The practical unit of resistance is the *ohm* and will be denoted by Ω. A linear resistor is therefore completely characterized by one number, its conductance or its resistance. If the value of the resistance is positive, the linear resistor is said to be a *positive resistor*. Otherwise, it is said to be a *negative resistor*. If $R = 0$, the linear resistor is said to be a *short circuit*. If $R = \infty$, it is said to be an *open circuit*.

[1] In view of the nonsymmetrical nature of this symbol, we may avoid drawing voltage polarity and current direction signs beside the symbol *provided* we agree to assume that the darkened edge is the negative terminal and that the current enters the positive terminal. This convention will be followed in this book.

It is important to differentiate between the terms *resistor* and *resistance;* the former refers to a black box, but the latter refers to a property associated with the black box.[1]

Exercise 1: Explain why it is unnecessary to differentiate between the terminals of the symbol for a linear resistor.

Exercise 2: A certain v-i curve is described by an equation $v = 10i + 5$. Is this a linear resistor?

1-6-2 NONLINEAR RESISTORS

If a resistor is characterized by a v-i curve other than a straight line through the origin, it is called a *nonlinear resistor.* In this case, the resistor can no longer be described by a single number, and hence the entire v-i curve must be given. This may be specified either graphically by a curve or analytically by a mathematical relationship. For example, consider the set of practical two-terminal resistors listed in Table 1-1.[2] Since these components are all commercially available, they have been given names and symbols.[3] Each resistor in this table is characterized graphically by a typical v-i curve usually supplied by the manufacturer. In some cases, it may be possible to derive a mathematical relationship which closely approximates a certain v-i curve. For example, from physical principles one can show that the v-i curve of a vacuum diode can be represented approximately by a $3/2$ power law, namely,[4]

$$i = kv^{3/2} \tag{1-13}$$

where k is a constant which depends on the physical dimensions of the internal structure of the diode. Similarly, a semiconductor junction diode can be represented approximately by an exponential law, namely,[5]

$$i = I_0(e^{kv} - 1) \tag{1-14}$$

where I_0 and k are constants which depend on the physical parameters of the diode. One can also sometimes derive an equation which approximates a v-i curve by interpolation and approximation techniques (see Appendix A). For example, the varistor shown in Table 1-1 can be represented approximately by the equation

$$v = \alpha i^{\beta} \tag{1-15}$$

[1] This subtle difference is not universally recognized. In many books, the terms resistor and resistance are used synonymously. In this book, the term resistance refers only to a linear resistor.

[2] To conform with the IEEE standard letter symbols for semiconductor devices (*IEEE Trans. Electron Devices,* vol. Ed-11, no. 8, pp. 392–397), we have chosen the *uppercase letters* V and I in favor of the *lowercase letters* v and i as used in the context. Whenever applicable, we shall also follow the latest IEEE standards for graphic symbols.

[3] In view of its relatively recent origin, the name and symbol for the constant-current diode are not universally used. The same device is sometimes referred to as a *current-limiting diode, a currector, a field-effect diode,* etc. A further discrepancy may be found in that portion of the v-i curve for negative voltages. Depending on how the device is made, the v-i curve for $v < 0$ either approximates an open circuit (horizontal line), *as will be assumed throughout this book,* or a short circuit (vertical line). Fortunately, this discrepancy is usually not important because, as will be shown later, only the portion of the v-i curve in the first quadrant is actually of practical interest. However, in any case, if the v-i curve for $v < 0$ approximates a short circuit, it can always be transformed into the v-i curve shown in Table 1-1 by connecting a junction diode in series

where α and β are constants which can be determined numerically from the curve. In all cases, we must remember that any mathematical relationship is at best an approximation to the actual v-i curve. Moreover, most v-i curves cannot be represented by such simple expressions as those given above. Therefore, the most general and common method to specify element characteristics is to describe the curve in graphical form.

1-6-3 CLASSIFICATION OF v-i CURVES

In order to be able to use the nonlinear resistors effectively in a practical design, it is necessary to classify v-i curves into various categories. For example, the v-i curves of the first three resistors in Table 1-1 have one property in common; namely, for each pair of points (v_1,i_1) and (v_2,i_2) on the curve, we observe that whenever $v_1 > v_2$, then $i_1 > i_2$. Such elements are said to be *strictly monotonically increasing* resistors. An examination of the v-i curves of the zener diode and the constant-current diode shows that they are not strictly monotonically increasing because if we pick a pair of points with voltages $v_1 > v_2$ along the horizontal portions of the v-i curve, then $i(v_1) \not> i(v_2)$. However, these v-i curves have another common property; namely, $i(v_1) \geq i(v_2)$ for any $v_1 > v_2$. Such elements are said to be monotonically (but not strictly) increasing resistors. The v-i curves of the tunnel diode and the remaining resistors below it are not monotonically increasing because each v-i curve has a portion having *negative slopes* ($di/dv < 0$). Such elements are sometimes called *negative-resistance elements*. Another common characteristic of a negative-resistance element is that either the voltage is a multivalued function of current (more than one voltage corresponds to some given value of current) or the current is a multivalued function of voltage (more than one current corresponds to some given value of voltage). In the first case, the current is a single-valued function of the voltage (but not vice versa); that is,

$$i = i(v) \tag{1-16}$$

and is therefore called a *voltage-controlled resistor*. In the second case, it is the voltage which is a single-valued function of current; that is,

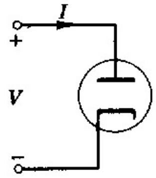
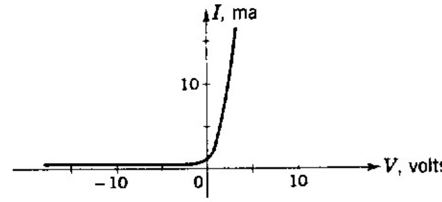
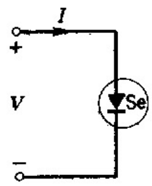
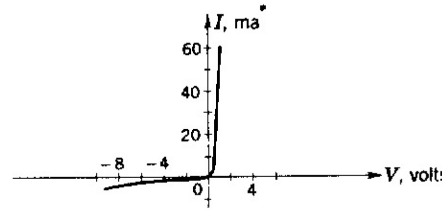
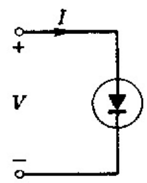
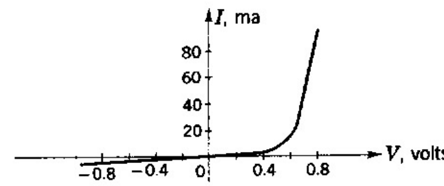
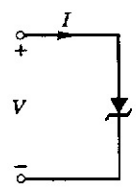
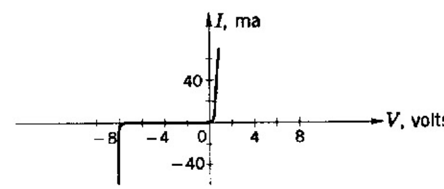
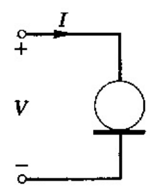
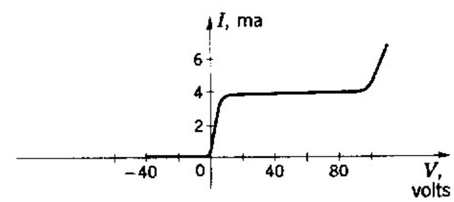
$$v = v(i) \tag{1-17}$$

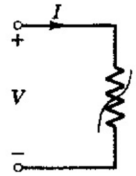
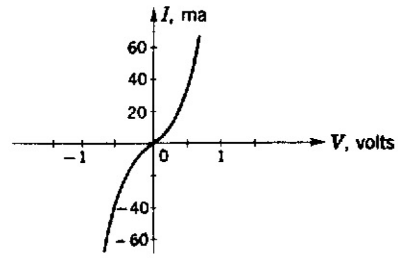
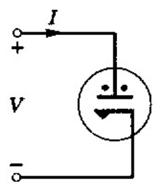
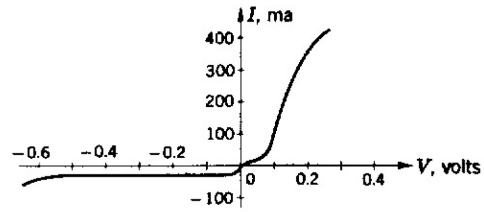
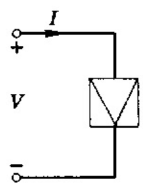
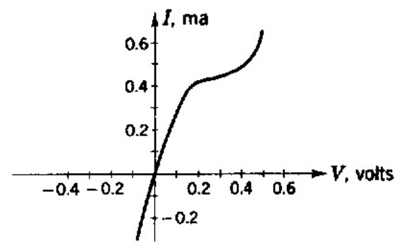
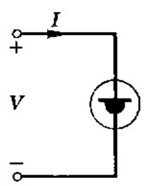
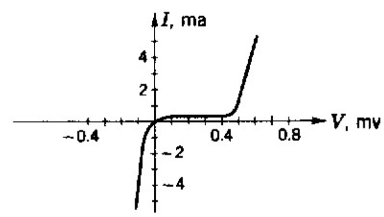
with the constant-current diode. For more information concerning this device, the reader is referred to J. M. Carroll, "Microelectronic Circuits and Applications," pp. 234 and 235, McGraw-Hill Book Company, New York, 1965; and J. M. Carroll, "Tunnel-Diode and Semiconductor Circuits," pp. 122–128, McGraw-Hill Book Company, New York, 1963.

[4] J. Langmuir, The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum, *Phys. Rev.*, vol. 2, pp. 450–486, 1913.

[5] J. F. Gibbons, "Semiconductor Electronics," McGraw-Hill Book Company, New York, 1966.

TABLE 1-1 *Practical two-terminal resistors.*

Name	Symbol	v-i characteristic curve
Vacuum diode		
Selenium diode		
Semiconductor (junction) diode		
Zener (avalanche, breakdown) diode		
Constant-current diode		

Name	Symbol	v-i characteristic curve
Varistor		
Solion liquid diode		
Tunnel resistor		
Back diode		
Tunnel diode