

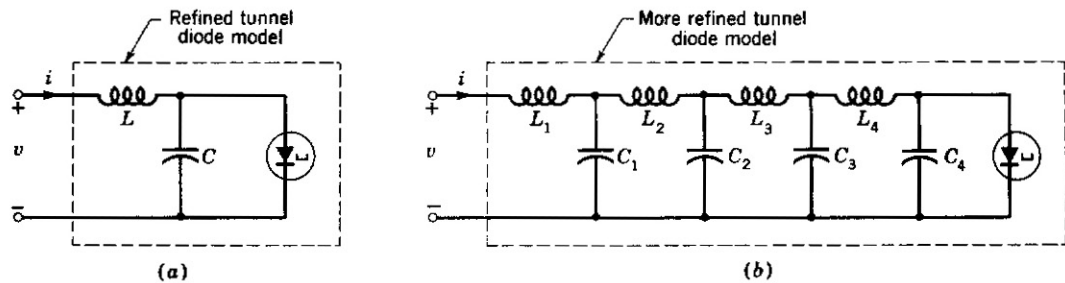
trajectories is tolerable, the model serves the purpose. Of course, in their desire to simplify analysis, engineers are often tempted to overidealize the model by stripping away some essential factors. In this case, the predicted solution may not be satisfactory.

The point we are driving at is that as engineers, we analyze the model which approximates an actual system. A model is always an idealization of a physical system. The more complex the model, the more accurate will the predicted solution be. Unfortunately, the analysis will also become more complicated. Hence, *a model is always a compromise between reality and simplicity.*

In the light of the above discussion, our definitions of a resistor, capacitor, and inductor must also be interpreted as models representing a physical device. For example, the v - i curve of the tunnel diode shown in Table 1-1 is a good model of a physical tunnel diode so long as the frequency at which we are operating is not very high. However, as the frequency increases, the static characteristic becomes less accurate, and a more realistic model must be found. For example, at very high frequencies, the connecting wires begin to behave like an inductance, and the capacitance between the wires gradually becomes significant. These elements are called "parasitic" or "stray" elements because they are invariably present, even though they are undesirable. A more realistic tunnel diode model must therefore include the parasitic elements such as the refined model shown in Fig. 1-26a. As the frequency gets higher, a still more complicated model such as shown in Fig. 1-26b may be chosen.

In this book, we shall be primarily interested in low-frequency models. In Chap. 11 we shall learn some basic techniques for constructing models of three-terminal devices in terms of two-terminal models. These low-frequency models can usually be refined for high-frequency analysis upon inclusion of appropriate parasitic elements.

Fig. 1-26. The static model of a tunnel diode must be refined for high-frequency analysis by the inclusion of appropriate parasitic inductances and capacitances at appropriate locations.



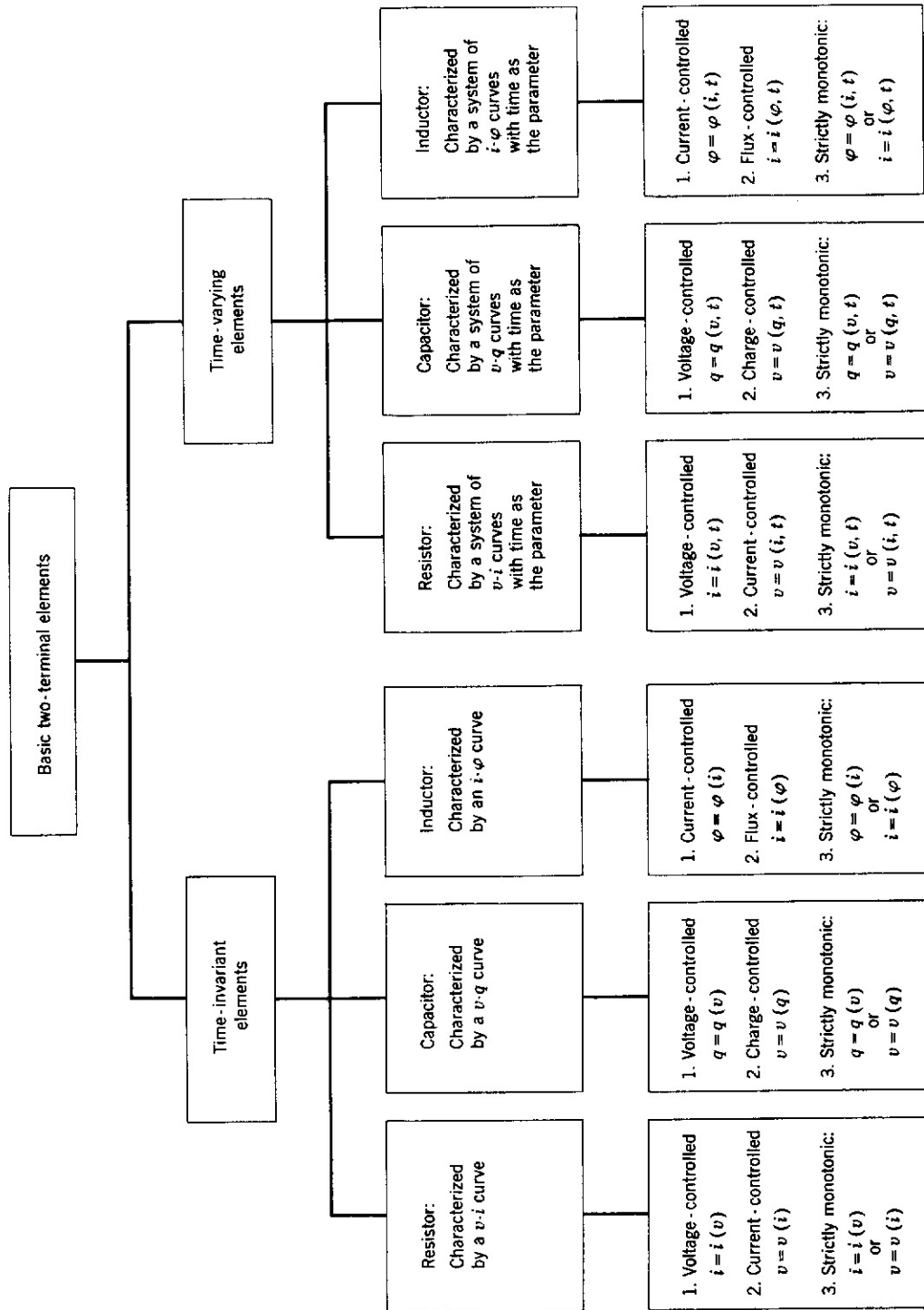


Fig. 1-27.

It stores *electric* energy and may be returned to its external circuit ($P_{C_{av}} = 0$). A flux-controlled inductor does not dissipate energy. It stores *magnetic* energy and may be returned to its external circuit ($P_{L_{av}} = 0$).

Basic two-terminal elements (See Fig. 1-27.)

PROBLEMS

- 1-1 In order to demonstrate the importance of reference direction and polarity, consider the following:
- Sketch the v - i curve of each of the two-terminal black boxes shown in Fig. P1-1a to e. (The element inside the black box is a tunnel diode whose V - I curve is given in Table 1-1.)
 - Give a simple rule for sketching the v - i curve of a resistor whose terminals have been reversed as in Fig. P1-1e.
 - There exists a certain class of nonlinear elements in which it is unnecessary to distinguish between the two terminals. Such elements are called *bilateral elements*. All other elements are said to be *nonbilateral*. Give an example of a bilateral and a nonbilateral resistor, inductor, and capacitor.
 - Find the necessary and sufficient condition for a nonlinear resistor, capacitor, or inductor to be bilateral.

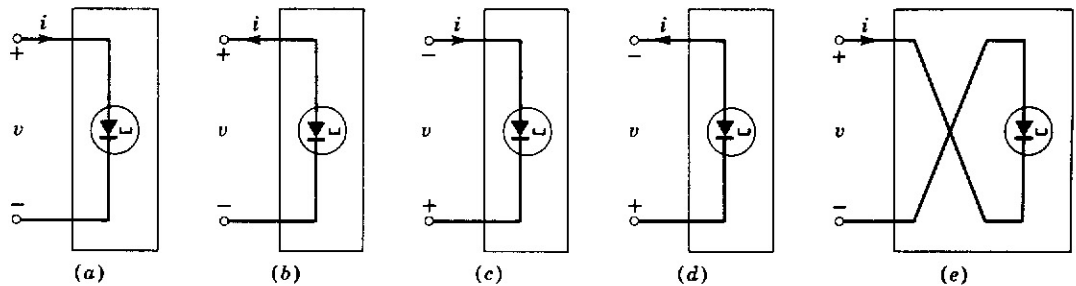


Fig. P1-1.

- 1-2 Sketch the v - i curve of each of the two-terminal black boxes shown in Fig. P1-2a to h. Refer to Table 1-1 for the V - I curves of the corresponding elements.

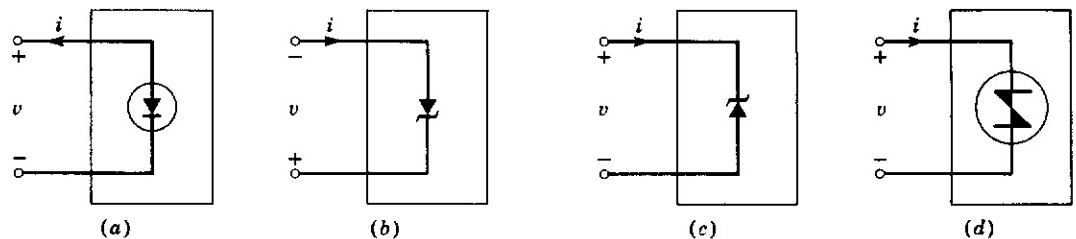


Fig. P1-2.

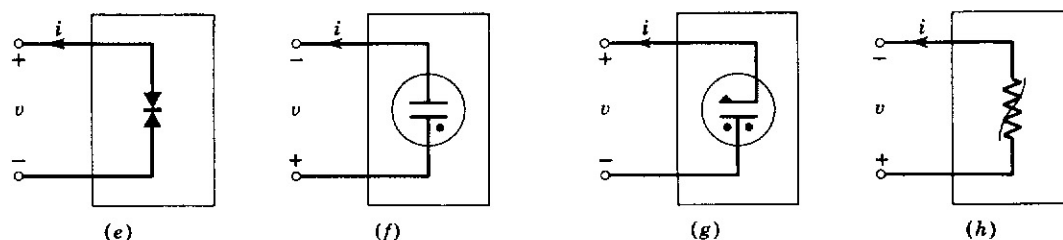


Fig. P1-2 (Continued).

1-3 The most accurate method for determining the v - i curve of a nonlinear resistor in the laboratory is the point-by-point method. Each point (E, I) on the curve is obtained by applying a voltage $v = E$ across the resistor and measuring the resulting current $i = I$. However, this method is rather tedious, and for most practical purposes it is desirable to design a v - i "curve tracer" that can display the v - i curve directly on an oscilloscope.

- (a) Devise a simple circuit to carry out the above task using the ordinary 60-Hz ac line voltage and a Variac (variable voltage transformer) to provide the desired range of input voltage required by the given resistor. You may use a linear resistor whose voltage drop can be used to sense the magnitude of the current in the nonlinear resistor. To avoid grounding problems, you may use a 1:1 isolation transformer.
- (b) Suppose that instead of using the line voltage as the energy source, we use the output voltage from a certain signal generator whose frequency can be changed from 10 Hz to 100 MHz. Do you expect the same v - i curve to be traced on the scope at all frequencies? If not, what frequency range must be chosen so that the v - i curve will agree approximately with the static curves supplied by the manufacturer?

1-4 It is sometimes convenient to define the dc resistance R_{dc} and the ac resistance R_{ac} at each point P of a v - i curve by

$$R_{dc} = \left. \frac{v}{i} \right|_P$$

$$R_{ac} = \left. \frac{dv}{di} \right|_P$$

- (a) Show that $R_{dc} = \cot \alpha$, where α is the angle between the v axis and the straight line from the origin to point P .
- (b) Show that $R_{ac} = \cot \beta$, where β is the angle between the v axis and the straight line tangent at point P .
- (c) Verify that for a passive resistor, the value of R_{dc} is always positive.
- (d) Verify that the value of R_{ac} may be either negative, zero, positive, or even infinite for a passive resistor.

- (e) Sketch the relationships R_{dc} versus v and R_{ac} versus v for the varistor type 1NXX5, the zener diode type 1NXX3, and the tunnel diode type 1NXX6. See Appendix D for the v - i curve of these elements.
- (f) Repeat (e) for the relationships R_{dc} versus i and R_{ac} versus i .
- 1-5 Consider the definitions of the dc resistance R_{dc} and ac resistance R_{ac} given in Problem 1-4.
- (a) If the resistor v - i curve is strictly monotonically increasing, what can you say about the curves R_{dc} versus v , R_{dc} versus i , R_{ac} versus v , and R_{ac} versus i ? Are they monotonic, single-valued, or multivalued?
- (b) Repeat (a) for a voltage-controlled resistor.
- (c) Repeat (a) for a current-controlled resistor.
- 1-6 It is sometimes convenient to define the dc conductance G_{dc} and the ac conductance G_{ac} at each point P of a v - i curve by
- $$G_{dc} = \left. \frac{i}{v} \right|_P$$
- $$G_{ac} = \left. \frac{di}{dv} \right|_P$$
- (a) Show that $G_{dc} = \tan \alpha$, where α is the angle between the v axis and the straight line from the origin to point P .
- (b) Show that $G_{ac} = \tan \beta$, where β is the angle between the v axis and the straight line tangent at point P .
- (c) Verify that for a passive resistor, the value of G_{dc} is always a finite, positive number.
- (d) Verify that the value of G_{ac} may be either negative, zero, positive, or even infinite for a passive resistor.
- (e) Sketch the relationships G_{dc} versus i and G_{ac} versus i for the varistor type 1NXX5, the zener diode type 1NXX3, and the tunnel diode type 1NXX6. See Appendix D for the v - i curve of these elements.
- (f) Repeat (e) for the relationships G_{dc} versus v and G_{ac} versus v .
- 1-7 Consider the definitions of the dc conductance G_{dc} and ac conductance G_{ac} given in Prob. 1-6.
- (a) If the resistor v - i curve is strictly monotonically increasing, what can you say about the curves G_{dc} versus i , G_{dc} versus v , G_{ac} versus i , and G_{ac} versus v ? Are they monotonic, single-valued, or multivalued?
- (b) Repeat (a) for a voltage-controlled resistor.
- (c) Repeat (a) for a current-controlled resistor.
- 1-8 If a sinusoidal voltage waveform $v = A \sin \omega t$ is applied across a nonlinear resistor characterized by a polynomial
- $$i = a_0 + a_1 v + a_2 v^2 + \cdots + a_n v^n$$

the resulting current will contain, in addition to the fundamental frequency term $i = B \sin \omega t$, other higher harmonic terms. In many practical applications, these harmonic terms are usually filtered out, in which case it becomes meaningful to define the ratio between the amplitudes of the fundamental voltage and current components to be the average resistance R_{av} .

- Find the average resistance R_{av} with $n = 3$.
- Assuming $a_0 = a_1 = a_2 = a_3 = 1$, plot R_{av} versus the amplitude A .
- What is the significance of the R_{av} -vs.- A curve obtained in (b)?

- 1-9 If a sinusoidal voltage waveform $v = A \cos \omega t$ is applied across a voltage-controlled capacitor characterized by

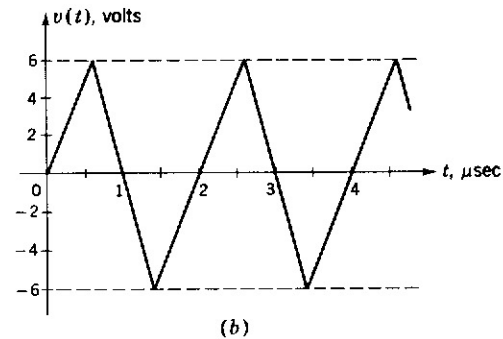
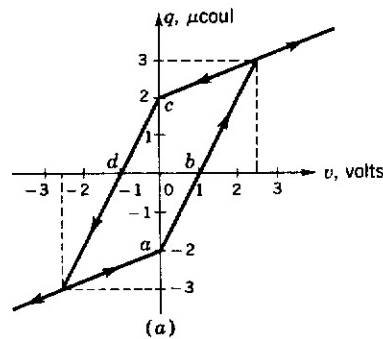
$$q = a_0 + a_1 v + a_2 v^2 + \cdots + a_n v^n$$

the resulting current will contain, in addition to the fundamental frequency term $i = B \sin \omega t$, other higher harmonic terms. In many practical applications these harmonic terms are filtered out, in which case the ratio between the amplitudes of the fundamental voltage and current components is usually called the *describing function* Z_C .

- Find the describing function Z_C with $n = 3$.
- Observe that unlike the average resistance in Prob. 1-8, the describing function Z_C of a capacitor is a function of the frequency ω . Assuming $a_0 = a_1 = a_2 = a_3 = 1$, plot the curve Z_C versus ω .
- What is the significance of the Z_C -vs.- ω curve?

- 1-10 The v - q curve of a practical, nonlinear capacitor with a barium titanate dielectric is shown in Fig. P1-10a. If the triangular voltage signal shown in Fig. P1-10b is applied across this capacitor, find the current waveform $i(t)$. Assume that the capacitor is operating initially at point a of the hysteresis curve. Assume also that the locus must follow the arrow directions.

Fig. P1-10.

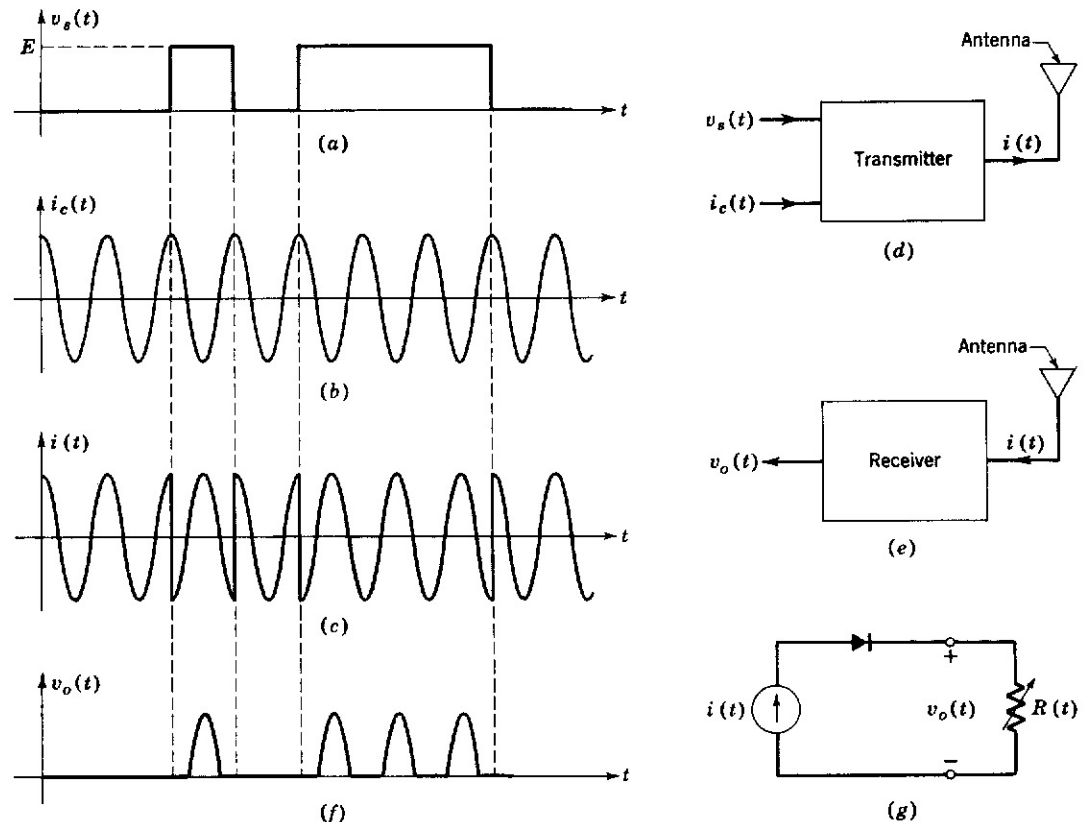


where $\sin \omega t$ and $\cos \omega t$ are very high-frequency (say 10 MHz) sine waves. This is the signal that will be received in Paris. Our problem is to recover $f_1(t)$ and $f_2(t)$ at the receiving end.

- (a) Show that $f_1(t)$ can be recovered by applying $i(t)$ to a time-varying linear resistor with $R(t) = 1 + \sin \omega t$ and then suppressing the components with a frequency higher than ω by means of a filter.
- (b) By a similar procedure, $f_2(t)$ can be recovered. Find the appropriate time-varying resistance $R(t)$ for accomplishing this.

1-14 One method for transmitting a telegraph signal over long distances is to modulate the "phase" of a high-frequency sinusoidal signal called the *carrier*. To be specific, suppose we wish to transmit the letter A in morse code by closing and opening the telegraph key at appropriate intervals. Corresponding to this code, the output voltage $v_s(t)$ shown in Fig. P1-14a will be generated. In order to transmit this waveform over long distances, an apparatus can be

Fig. P1-14.



designed to change the phase of the carrier signal $i_c(t)$ shown in Fig. P1-14b abruptly by 180° each time $v_s(t)$ changes its amplitude. For example, the resulting current waveform $i(t)$ is shown in Fig. P1-14c. This is the signal being transmitted and received, as shown in Fig. P1-14d and e. Our problem is to decode the received current waveform $i(t)$ so as to recover the message A. This can be accomplished by applying $i(t)$ (as a current source) to the time-varying circuit shown in Fig. P1-14g so as to produce the output voltage $v_o(t)$ shown in Fig. P1-14f. Observe that even though $v_o(t)$ is not identical with $v_s(t)$, the nature of the waveform is unmistakably similar to that of Fig. P1-14g. Hence the above decoding scheme would accomplish our objective.

- (a) Specify the time-varying resistance $R(t)$ for accomplishing this task.
- (b) What can you say about the frequency of $R(t)$ in comparison with that of the carrier signal $i_c(t)$?
- (c) The above scheme for decoding the signal is known as the *synchronous phase detection*. Explain why.